Washington State

6 - 12

Includes
Mathematics 1, 2, and 3

Mathematics

Learning Standards

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SUPERINTENDENT OF PUBLIC INSTRUCTION
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Introduction

Overview

The *Washington State K–12 Mathematics Standards* outline the mathematics learning expectations for all students in Washington. These standards describe the mathematics content, procedures, applications, and processes that students are expected to learn. The topics and mathematical strands represented across grades K–12 constitute a mathematically complete program that includes the study of numbers, operations, geometry, measurement, algebra, data analysis, and important mathematical processes.

Organization of the standards

The *Washington State K–12 Mathematics Standards* are organized by grade level for grades K–8 and by course for grades 9–12, with each grade/course consisting of three elements: *Core Content*, *Additional Key Content*, and *Core Processes*. Each of these elements contains *Performance Expectations* and *Explanatory Comments and Examples*.

Core Content areas describe the major mathematical focuses of each grade level or course. A limited number of priorities for each grade level in grades K–8 and for each high school course are identified, so teachers know which topics call for the most time and emphasis. Each priority area includes a descriptive paragraph that highlights the mathematics addressed and its role in a student's overall mathematics learning.

Additional Key Content contains important expectations that do not warrant the same amount of instructional time as the Core Content areas. These are expectations that might extend a previously learned skill, plant a seed for future development, or address a focused topic, such as scientific notation. Although they need less classroom time, these expectations are important, are expected to be taught, and may be assessed as part of Washington State's assessment system. The content in this section allows students to build a coherent knowledge of mathematics from year to year.

Core Processes include expectations that address reasoning, problem solving, and communication. While these processes are incorporated throughout other content expectations, they are presented in this section to clearly describe the breadth and scope of what is expected in each grade or course. In Core Processes, at least two rich problems that cut across Core or Key Content areas are included as examples for each grade or course. These problems illustrate the types and breadth of problems that could be used in the classroom.

Performance Expectations, in keeping with the accepted definition of standards, describe what students should know and be able to do at each grade level. These statements are the core of the document. They are designed to provide clear guidance to teachers about the mathematics that is to be taught and learned.

Explanatory Comments and Examples accompany most of the expectations. These are not technically performance expectations. However, taken together with the Performance Expectations, they provide a full context and clear understanding of the expectation.

The comments expand upon the meaning of the expectations. Explanatory text might clarify the parameters regarding the type or size of numbers, provide more information about student expectations regarding mathematical understanding, or give expanded detail to mathematical definitions, laws, principles, and forms included in the expectation.

The example problems include those that are typical of the problems students should do, those that illustrate various types of problems associated with a particular performance expectation, and those that illustrate the expected limits of difficulty for problems related to a performance expectation. Teachers are not expected to teach these particular examples or to limit what they teach to these examples. Teachers and quality instructional materials will incorporate many different types of examples that support the teaching of the content described in any expectation.

In some instances, comments related to pedagogy are included in the standards as familiar illustrations to the teacher. Teachers are not expected to use these particular teaching methods or to limit the methods they use to the methods included in the document. These, too, are illustrative, showing one way an expectation might be taught.

Although, technically, the performance expectations set the requirements for Washington students, people will consider the entire document as the Washington mathematics standards. Thus, the term *standards*, as used here, refers to the complete set of Performance Expectations, Explanatory Comments and Examples, Core Content, Additional Key Content, and Core Processes. Making sense of the standards from any grade level or course calls for understanding the interplay of Core Content, Additional Key Content, and Core Processes for that grade or course.

What standards are not

Performance expectations do not describe how the mathematics will be taught. Decisions about instructional methods and materials are left to professional teachers who are knowledgeable about the mathematics being taught and about the needs of their students.

The standards are not comprehensive. They do not describe everything that could be taught in a classroom. Teachers may choose to go beyond what is included in this document to provide related or supporting content. They should teach beyond the standards to those students ready for additional challenges. Standards related to number skills, in particular, should be viewed as a floor—minimum expectations—and not a ceiling. A student who can order and compare numbers to 120 should be given every opportunity to apply these concepts to larger numbers.

The standards are not test specifications. Excessive detail, such as the size of numbers that can be tested and the conditions for assessment, clouds the clarity and usability of a standards document, generally, and a performance expectation, specifically. For example, it is sufficient to say "Identify, describe, and classify triangles by angle measure and number of congruent sides," without specifying that acute, right, and obtuse are types of triangles classified by their angle size and that scalene, isosceles, and equilateral are types of triangles classified by their side length. Sometimes this type of information is included in the comments section, but generally this level of detail is left to other documents.

What about strands?

Many states' standards are organized around mathematical content strands—generally some combination of numbers, operations, geometry, measurement, algebra, and data/statistics. However, the *Washington State K–12 Mathematics Standards* are organized according to the priorities described as Core Content rather than being organized in strands. Nevertheless, it is still useful to know what content strands are addressed in particular Core Content and Additional Key Content areas. Thus, mathematics content strands are identified in parentheses at the beginning of each Core Content or Additional Key Content area. Five content strands have been identified for this purpose: Numbers, Operations, Geometry/Measurement, Algebra, and Data/Statistics/Probability. For each of these strands, a separate K–12 strand document allows teachers and other readers to track the development of knowledge and skills across grades and courses. An additional strand document on the Core Processes tracks the development of reasoning, problem solving, and communication across grades K–12.

A well-balanced mathematics program for all students

An effective mathematics program balances three important components of mathematics—conceptual understanding (making sense of mathematics), procedural proficiency (skills, facts, and procedures), and problem solving and mathematical processes (using mathematics to reason, think, and apply mathematical knowledge). These standards make clear the importance of all three of these components, purposefully interwoven to support students' development as increasingly sophisticated mathematical thinkers. The standards are written to support the development of students so that they know and understand mathematics.

Conceptual understanding (making sense of mathematics)
Students who understand a concept are able to identify examples as well as non-examples, describe the concept (for example, with words, symbols, drawings, tables, or models), provide a definition of the concept, and use the concept in different ways. Conceptual understanding is woven throughout these standards. Expectations with verbs like *demonstrate*, *describe*, *represent*, *connect*, and *justify*, for example, ask students to show their understanding. Furthermore, expectations addressing both procedures and applications often ask students to connect their conceptual understanding to the procedures being learned or problems being solved.

Procedural proficiency (skills, facts, and procedures)
Learning basic facts is important for developing mathematical understanding. In these standards, clear expectations address students' knowledge of basic facts. The use of the term *basic facts* typically encompasses addition and multiplication facts up to and including 10 + 10 and 10 x 10 and their related subtraction and division facts. In these standards, students are expected to "quickly recall" basic facts. "Quickly recall" means that the student has ready and effective access to facts without having to go through a development process or strategy, such as counting up or drawing a picture, every time he or she needs to know a fact. Simply put, students need to know their basic facts.

Building on a sound conceptual understanding of addition, subtraction, multiplication, and division, Washington's standards include a specific discussion of students' need to understand and use the standard algorithms generally seen in the United States to add, subtract, multiply, and divide whole numbers. There are other possible algorithms students might also use to perform these operations and some teachers may find value in students learning multiple algorithms to enhance understanding.

Algorithms are step-by-step mathematical procedures that, if followed correctly, always produce a correct solution or answer. Generalized procedures are used throughout mathematics, such as in drawing geometric constructions or going through the steps involved in solving an algebraic equation. Students should come to understand that mathematical procedures are a useful and important part of mathematics.

The term *fluency* is used in these standards to describe the expected level and depth of a student's knowledge of a computational procedure. For the purposes of these standards, a student is considered fluent when the procedure can be performed immediately and accurately. Also, when fluent, the student knows when it is appropriate to use a particular procedure in a problem or situation. A student who is fluent in a procedure has a tool that can be applied reflexively and doesn't distract from the task of solving the problem at hand. The procedure is stored in long-term memory, leaving working memory available to focus on the problem.

Problem solving and mathematical processes (reasoning and thinking to apply mathematical content)

Mathematical processes, including reasoning, problem solving, and communication, are essential in a well-balanced mathematics program. Students must be able to reason, solve problems, and communicate their understanding in effective ways. While it is impossible to completely separate processes and content, the standards' explicit description of processes at each grade level calls attention to their importance within a well-balanced mathematics program. Some common language is used to describe the Core Processes across the grades and within grade bands (K–2, 3–5, 6–8, and 9–12). The problems students will address, as well as the language and symbolism they will use to communicate their mathematical understanding, become more sophisticated from grade to grade. These shifts across the grades reflect the increasing complexity of content and the increasing rigor as students deal with more challenging problems, much in the same way that reading skills develop from grade to grade with increasingly complex reading material.

Technology

The role of technology in learning mathematics is a complex issue, because of the ever-changing capabilities of technological tools, differing beliefs in the contributions of technology to a student's education, and equitable student access to tools. However, one principle remains constant: The focus of mathematics instruction should always be on the mathematics to be learned and on helping students learn that mathematics.

Technology ***should be used*** *when it supports the mathematics to be learned, and technology* ***should not be used*** *when it might interfere with learning.*

Calculators and other technological tools, such as computer algebra systems, dynamic geometry software, applets, spreadsheets, and interactive presentation devices are an important part of today's classroom. But the use of technology cannot replace conceptual understanding, computational fluency, or problem-solving skills.

Washington's standards make clear that some performance expectations are to be done without the aid of technology. Elementary students are expected to know facts and basic computational procedures without using a calculator. At the secondary level, students should compute with polynomials, solve equations, sketch simple graphs, and perform some constructions without the use of technology. Students should continue to use previously learned facts and skills in subsequent grade levels to maintain their fluency without the assistance of a calculator.

At the elementary level, calculators are less useful than they will be in later grades. The core of elementary school—number sense and computational fluency—does not require a calculator. However, this is not to say that students couldn't use calculators to investigate mathematical situations and to solve problems involving complicated numbers, lots of numbers, or data sets.

As middle school students deal with increasingly complex statistical data and represent proportional relationships with graphs and tables, a calculator or technological tool with these functions can be useful for representing relationships in multiple ways. At the high school level, graphing calculators become valuable tools as all students tackle the challenges of algebra and geometry to prepare for a range of postsecondary options in a technological world. Graphing calculators and spreadsheets allow students to explore and solve problems with classes of functions in ways that were previously impossible.

While the majority of performance expectations describe skills and knowledge that a student could demonstrate without technology, learning when it is helpful to use these tools and when it is cumbersome is part of becoming mathematically literate. When students become dependent upon technology to solve basic math problems, the focus of mathematics instruction to help students learn mathematics has failed.

Connecting to the Washington Essential Academic Learning Requirements (EALRs) and Grade Level Expectations (GLEs)

The new *Washington State K–12 Mathematics Standards* continue Washington's longstanding commitment to teaching mathematics content and mathematical thinking. The new standards replace the former Essential Academic Learning Requirements (EALRs) and Grade Level Expectations (GLEs). The former mathematics EALRs, listed below, represent threads in the mathematical content, reasoning, problem solving, and communication that are reflected in these new standards.

EALR 1: The student understands and applies the concepts and procedures of mathematics.

EALR 2: The student uses mathematics to define and solve problems.

EALR 3: The student uses mathematical reasoning.

EALR 4: The student communicates knowledge and understanding in both everyday and mathematical language.

EALR 5: The student understands how mathematical ideas connect within mathematics, to other subjects.

System-wide standards implementation activities

These mathematics standards represent an important step in ramping up mathematics teaching and learning in the state. The standards provide a critical foundation, but are only the first step. Their success will depend on the implementation efforts that match many of the activities outlined in Washington's Joint Mathematics Action Plan. This includes attention to:

- Aligning the Washington Assessment for Student Learning to these standards;
- Identifying mathematics curriculum and instructional support materials;
- Providing systematic professional development so that instruction aligns with the standards;
- Developing online availability of the standards in various forms and formats, with additional example problems, classroom activities, and possible lessons embedded.

As with any comprehensive initiative, fully implementing these standards will not occur overnight. This implementation process will take time, as teachers become more familiar with the standards and as students enter each grade having learned more of the standards from previous grades. There is always a tension of balancing the need to raise the bar with the reality of how much change is possible, and how quickly this change can be implemented in real schools with real teachers and real students.

Change is hard. These standards expect more of students and more of their teachers. Still, if Washington's students are to be prepared to be competitive and to reach their highest potential, implementing these standards will pay off for years to come.

GRADE 6
STANDARDS

SUPERINTENDENT OF PUBLIC INSTRUCTION
WASHINGTON

Grade 6

6.1. Core Content: *Multiplication and division of fractions and decimals* *(Numbers, Operations, Algebra)*

Students have done extensive work with fractions and decimals in previous grades and are now prepared to learn how to multiply and divide fractions and decimals with understanding. They can solve a wide variety of problems that involve the numbers they see every day—whole numbers, fractions, and decimals. By using approximations of fractions and decimals, students estimate computations and verify that their answers make sense.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
6.1.A Compare and order non-negative fractions, decimals, and integers using the number line, lists, and the symbols <, >, or =.	Examples: • List the numbers $2\frac{1}{3}$, $\frac{4}{5}$, 0.94, $\frac{5}{4}$, 1.1, and $\frac{43}{50}$ in increasing order, and then graph the numbers on the number line. • Compare each pair of numbers using <, >, or =. $\frac{4}{5}$ $\square$ 1.2 $\frac{7}{4}$ $\square$ $1\frac{3}{4}$ $2\frac{7}{8}$ $\square$ 2.5
6.1.B Represent multiplication and division of non-negative fractions and decimals using area models and the number line, and connect each representation to the related equation.	This expectation addresses the conceptual meaning of multiplication and division of fractions and decimals. Students should be familiar with the use of visual representations like pictures (e.g., sketching the problem, grid paper) and physical objects (e.g., tangrams, cuisenaire rods). They should connect the visual representation to the corresponding equation. The procedures for multiplying fractions and decimals are addressed in 6.1.D and 6.1.E.
6.1.C Estimate products and quotients of fractions and decimals.	Example: • $0.28 \div 0.96 \approx 0.3 \div 1$; $0.3 \div 1 = 0.3$ $0.24 \times 12.4 \approx \frac{1}{4} \times 12.4$; $\frac{1}{4} \times 12.4 = 3.1$ $\frac{3}{13} \times \frac{20}{41} \approx \frac{1}{4} \times \frac{1}{2}$; $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
6.1.D Fluently and accurately multiply and divide non-negative fractions and explain the inverse relationship between multiplication and division with fractions.	Students should understand the inverse relationship between multiplication and division, developed in grade three and now extended to fractions. Students should work with different types of rational numbers, including whole numbers and mixed numbers, as they continue to expand their understanding of the set of rational numbers. Example: • Multiply or divide. $\frac{4}{5} \times \frac{2}{3}$ $\quad 6 \div \frac{3}{8}$ $2\frac{1}{4} \times 3\frac{1}{2}$ $\quad 4\frac{1}{5} \div 1\frac{2}{3}$
6.1.E Multiply and divide whole numbers and decimals by 1000, 100, 10, 1, 0.1, 0.01, and 0.001.	This expectation extends what students know about the place value system and about multiplication and division and expands their set of mental math tools. As students work with multiplication by these powers of 10, they can gain an understanding of how numbers relate to each other based on their relative sizes. Example: • Mentally compute 0.01 x 435.
6.1.F Fluently and accurately multiply and divide non-negative decimals.	Students should understand the inverse relationship between multiplication and division, developed in grade three and now extended to decimals. Students should work with different types of decimals, including decimals greater than 1, decimals less than 1, and whole numbers, as they continue to expand their understanding of the set of rational numbers. Example: • Multiply or divide. 0.84×1.5 $\quad 2.04 \times 32$ $7.85 \div 0.32$ $\quad 17.28 \div 1.2$
6.1.G Describe the effect of multiplying or dividing a number by one, by zero, by a number between zero and one, and by a number greater than one.	Examples: • Without doing any computation, list 74, 0.43×74, and $74 \div 0.85$ in increasing order and explain your reasoning. • Explain why $\frac{4}{0}$ is undefined.

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
6.1.H Solve single- and multi-step word problems involving operations with fractions and decimals and verify the solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations. Example: • Every day has 24 hours. Ali sleeps 3/8 of the day. Dawson sleeps 1/3 of the day. Maddie sleeps 7.2 hours in a day. Who sleeps the longest? By how much?

Grade 6

***6.2. Core Content:** Mathematical expressions and equations* *(Operations, Algebra)*

Students continue to develop their understanding of how letters are used to represent numbers in mathematics—an important foundation for algebraic thinking. Students use tables, words, numbers, graphs, and equations to describe simple linear relationships. They write and evaluate expressions and write and solve equations. By developing these algebraic skills at the middle school level, students will be able to make a smooth transition to high school mathematics.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
6.2.A Write a mathematical expression or equation with variables to represent information in a table or given situation.	Examples: • What expression can be substituted for the question mark? x: 1, 2, 3, 4, …, x y: 2.5, 5, 7.5, 10, …, ? • A t-shirt printing company charges $7 for each t-shirt it prints. Write an equation that represents the total cost, *c*, for ordering a specific quantity, *t*, of these t-shirts.
6.2.B Draw a first-quadrant graph in the coordinate plane to represent information in a table or given situation.	Example: • Mikayla and her sister are making beaded bracelets to sell at a school craft fair. They can make two bracelets every 30 minutes. Draw a graph that represents the number of bracelets the girls will have made at any point during the 6 hours they work.
6.2.C Evaluate mathematical expressions when the value for each variable is given.	Examples: • Evaluate $2s + 5t$ when $s = 3.4$ and $t = 1.8$. • Evaluate $\frac{2}{3}x - 14$ when $x = 60$.
6.2.D Apply the commutative, associative, and distributive properties, and use the order of operations to evaluate mathematical expressions.	Examples: • Simplify $6\left(\frac{1}{2}+\frac{1}{3}\right)$, with and without the use of the distributive property. • Evaluate $b - 3(2a - 7)$ when $a = 5.4$ and $b = 31.7$.

The table in 6.2.A:

x	1	2	3	4	…	x
y	2.5	5	7.5	10	…	?

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
6.2.E Solve one-step equations and verify solutions.	Students solve equations using number sense, physical objects (e.g., balance scales), pictures, or properties of equality. Example: • Solve for the variable in each equation below. $112 = 7a$ $1.4y = 42$ $2\frac{1}{2} = b + \frac{1}{3}$ $\frac{y}{45} = \frac{7}{15}$
6.2.F Solve word problems using mathematical expressions and equations and verify solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations. Example: • Zane and his friends drove across the United States at an average speed of 55 mph. Write expressions to show how far they traveled in 12 hours, in 18 hours, and in *n* hours. How long did it take them to drive 1,430 miles? Verify your solution.

Grade 6

6.3. Core Content: *Ratios, rates, and percents* *(Numbers, Operations, Geometry/Measurement, Algebra, Data/Statistics/Probability)*

Students extend their knowledge of fractions to develop an understanding of what a ratio is and how it relates to a rate and a percent. Fractions, ratios, rates, and percents appear daily in the media and in everyday calculations like determining the sale price at a retail store or figuring out gas mileage. Students solve a variety of problems related to such situations. A solid understanding of ratios and rates is important for work involving proportional relationships in grade seven.

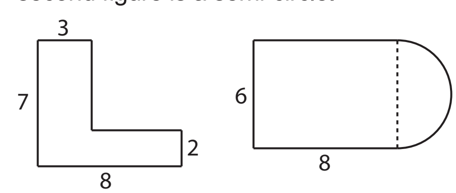
Performance Expectations *Students are expected to:*		***Explanatory Comments and Examples***
6.3.A	Identify and write ratios as comparisons of part-to-part and part-to-whole relationships.	Example: • If there are 10 boys and 12 girls in a class, what is the ratio of boys to girls? What is the ratio of the number of boys to the total number of students in the class?
6.3.B	Write ratios to represent a variety of rates.	Example: • Julio drove his car 579 miles and used 15 gallons of gasoline. How many miles per gallon did his car get during the trip? Explain your answer.
6.3.C	Represent percents visually and numerically, and convert between the fractional, decimal, and percent representations of a number.	In addition to general translations among these representations, this expectation includes the quick recall of equivalent forms of common fractions (with denominators like 2, 3, 4, 5, 8, and 10), decimals, and percents. It also includes the understanding that a fraction represents division, an important conceptual background for writing fractions as decimals. Examples: • Represent $\frac{75}{100}$ as a percent using numbers, a picture, and a circle graph. • Represent 40% as a fraction and as a decimal. • Write $\frac{13}{16}$ as a decimal and as a percent.

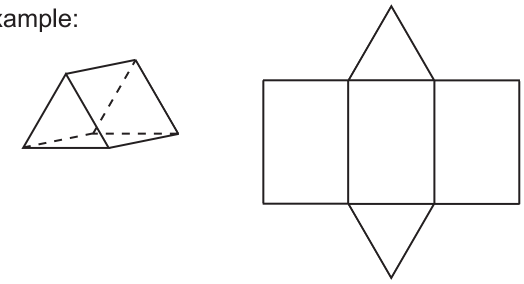
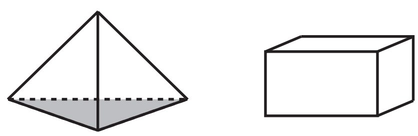
Performance Expectations *Students are expected to:*		*Explanatory Comments and Examples*
6.3.D	Solve single- and multi-step word problems involving ratios, rates, and percents, and verify the solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations. Examples: • An item is advertised as being 25% off the regular price. If the sale price is $42, what was the original regular price? Verify your solution. • Sally had a business meeting in a city 100 miles away. In the morning, she drove an average speed of 60 miles per hour, but in the evening when she returned, she averaged only 40 miles per hour. How much longer did the evening trip take than the morning trip? Explain your reasoning.
6.3.E	Identify the ratio of the circumference to the diameter of a circle as the constant π, and recognize $\frac{22}{7}$ and 3.14 as common approximations of π.	Example: • Measure the diameter and circumference of several circular objects. Divide each circumference by its diameter. What do you notice about the results?
6.3.F	Determine the experimental probability of a simple event using data collected in an experiment.	The term *experimental probability* refers here to the relative frequency that was observed in an experiment. Example: • Tim is checking the apples in his orchard for worms. Selecting apples at random, he finds 9 apples with worms and 63 apples without worms. What is the experimental probability that a given apple from his orchard has a worm in it?
6.3.G	Determine the theoretical probability of an event and its complement and represent the probability as a fraction or decimal from 0 to 1 or as a percent from 0 to 100.	Example: • A bag contains 4 green marbles, 6 red marbles, and 10 blue marbles. If one marble is drawn randomly from the bag, what is the probability it will be red? What is the probability that it will not be red?

Grade 6

6.4. Core Content: *Two- and three-dimensional figures* *(Geometry/Measurement, Algebra)*

Students extend what they know about area and perimeter to more complex two-dimensional figures, including circles. They find the surface area and volume of simple three-dimensional figures. As they learn about these important concepts, students can solve problems involving more complex figures than in earlier grades and use geometry to deal with a wider range of situations. These fundamental skills of geometry and measurement are increasingly called for in the workplace and they lead to a more formal study of geometry in high school.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
6.4.A Determine the circumference and area of circles.	Examples: • Determine the area of a circle with a diameter of 12 inches. • Determine the circumference of a circle with a radius of 32 centimeters.
6.4.B Determine the perimeter and area of a composite figure that can be divided into triangles, rectangles, and parts of circles.	Although students have worked with various quadrilaterals in the past, this expectation includes other quadrilaterals such as trapezoids or irregular quadrilaterals, as well as any other composite figure that can be divided into figures for which students have calculated areas before. Example: • Determine the area and perimeter of each of the following figures, assuming that the dimensions on the figures are in feet. The curved portion of the second figure is a semi-circle. 
6.4.C Solve single- and multi-step word problems involving the relationships among radius, diameter, circumference, and area of circles, and verify the solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations. Example: • Captain Jenkins determined that the distance around a circular island is 44 miles. What is the distance from the shore to the buried treasure in the center of the island? What is the area of the island?

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
6.4.D Recognize and draw two-dimensional representations of three-dimensional figures.	The net of a rectangular prism consists of six rectangles that can then be folded to make the prism. The net of a cylinder consists of two circles and a rectangle. Example: 
6.4.E Determine the surface area and volume of rectangular prisms using appropriate formulas and explain why the formulas work.	Students may determine surface area by calculating the area of the faces and adding the results.
6.4.F Determine the surface area of a pyramid.	
6.4.G Describe and sort polyhedra by their attributes: parallel faces, types of faces, number of faces, edges, and vertices.	Prisms and pyramids are the focus at this level. Examples: • How many pairs of parallel faces does each polyhedron have? Explain your answer.  • What type of polyhedron has two parallel triangular faces and three non-parallel rectangular faces?

Grade 6

6.5. Additional Key Content *(Numbers, Operations)*

Students extend their mental math skills now that they have learned all of the operations—addition, subtraction, multiplication, and division—with whole numbers, fractions, and decimals. Students continue to expand their understanding of our number system as they are introduced to negative numbers for describing positions or quantities below zero. These numbers are a critical foundation for algebra, and students will learn how to add, subtract, multiply, and divide positive and negative numbers in seventh grade as further preparation for algebraic study.

Performance Expectations

Students are expected to:

Explanatory Comments and Examples

6.5.A Use strategies for mental computations with non-negative whole numbers, fractions, and decimals.

Examples:

- John wants to find the total number of hours he worked this week. Use his time card below to find the total.

Days	Monday	Tuesday	Wednesday	Thursday	Friday
Days	$4\frac{1}{4}$	3	$6\frac{1}{2}$	$7\frac{1}{2}$	$1\frac{1}{2}$

- What is the total cost for items priced at \$25.99 and \$32.95? (A student may think of something like 25.99 + 32.95 = (26 + 33) – 0.06 = 58.94.)

6.5.B Locate positive and negative integers on the number line and use integers to represent quantities in various contexts.

Contexts could include elevation, temperature, or debt, among others.

6.5.C Compare and order positive and negative integers using the number line, lists, and the symbols <, >, or =.

Examples:

- Compare each pair of numbers using <, >, or =.

  -11 ☐ -14

  -7 ☐ 4

  -101 ☐ -94

Grade 6

6.6. Core Processes: *Reasoning, problem solving, and communication*

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher-level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade six, students solve problems that involve fractions and decimals as well as rates and ratios in preparation for studying proportional relationships and algebraic reasoning in grade seven.

Performance Expectations

Students are expected to:

6.6.A Analyze a problem situation to determine the question(s) to be answered.

6.6.B Identify relevant, missing, and extraneous information related to the solution to a problem.

6.6.C Analyze and compare mathematical strategies for solving problems, and select and use one or more strategies to solve a problem.

6.6.D Represent a problem situation, describe the process used to solve the problem, and verify the reasonableness of the solution.

6.6.E Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.

6.6.F Apply a previously used problem-solving strategy in a new context.

6.6.G Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.

6.6.H Make and test conjectures based on data (or information) collected from explorations and experiments.

Explanatory Comments and Examples

Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, physical objects, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Examples:

- As part of her exercise routine, Carmen jogs twice around the perimeter of a square park that measures $\frac{5}{8}$ mile on each side. On Monday, she started at one corner of the park and jogged $\frac{2}{3}$ of the way around in 17 minutes before stopping at a small pond in the park to feed some ducks. How far had Carmen run when she reached the pond? What percent of her planned total distance had Carmen completed when she stopped to feed the ducks? If it took Carmen 17 minutes to jog to the point where she stopped, assuming that she continued running in the same direction at the same pace and did not stop again, how long would it have taken her to get back to her starting point? Explain your answers.
- At Springhill Elementary School's annual fair, Vanessa is playing a game called "Find the Key." A key is randomly placed somewhere in one of the rooms shown on the map below. (The key cannot be placed in the hallway.)

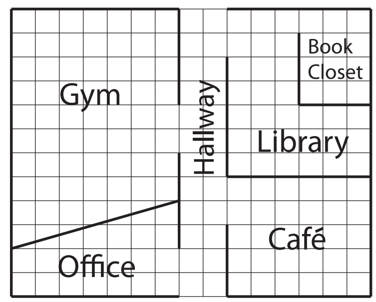
Performance Expectations

Students are expected to:

6.6 cont.

Explanatory Comments and Examples

To win the game, Vanessa must correctly guess the room where the key is placed. Use what you know about the sizes of the rooms to determine the probability that the key is placed in the gym, the office, the café, the book closet, or the library. Write each probability as a simplified fraction, a decimal, and a percent. Which room should Vanessa select in order to have the best chance of winning? Justify the solution.



GRADE 7
STANDARDS

Grade 7

7.1. Core Content: *Rational numbers and linear equations* *(Numbers, Operations, Algebra)*

Students add, subtract, multiply, and divide rational numbers—fractions, decimals, and integers—including both positive and negative numbers. With the inclusion of negative numbers, students can move more deeply into algebraic content that involves the full set of rational numbers. They also approach problems that deal with a wider range of contexts than before. Using generalized algebraic skills and approaches, students can approach a wide range of problems involving any type of rational number, adapting strategies for solving one problem to different problems in different settings with underlying similarities.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
7.1.A Compare and order rational numbers using the number line, lists, and the symbols <, >, or =.	Examples: • List the numbers $\frac{2}{3}$, $-\frac{2}{3}$, 1.2, $\frac{4}{3}$, $-\frac{4}{3}$, -1.2, and $-\frac{7}{4}$ in increasing order, and graph the numbers on the number line. • Compare each pair of numbers using <, >, or =. $\frac{-11}{20} \square \frac{-13}{21}$ $-\frac{7}{5} \square -1.35$ $-2\frac{3}{4} \square -2.75$
7.1.B Represent addition, subtraction, multiplication, and division of positive and negative integers visually and numerically.	Students should be familiar with the use of the number line and physical materials, such as colored chips, to represent computation with integers. They should connect numerical and physical representations to the computation. The procedures are addressed in 7.1.C. Examples: • Use a picture, words, or physical objects to illustrate 3 – 7; -3 – 7; -3 – (-7); (-3)(-7); 21 ÷ (-3). • At noon on a certain day, the temperature was 13°; at 10 p.m. the same day, the temperature was -8°. How many degrees did the temperature drop between noon and 10 p.m.?

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
7.1.C Fluently and accurately add, subtract, multiply, and divide rational numbers.	This expectation brings together what students know about the four operations with positive and negative numbers of all kinds—integers, fractions, and decimals. Some of these skills will have been recently learned and may need careful development and reinforcement. This is an opportunity to demonstrate connections among the operations and to show similarities and differences in the performance of these operations with different types of numbers. Visual representations may be helpful as students begin this work, and they may become less necessary as students become increasingly fluent with the operations. Examples: • $-\frac{4}{3} - \frac{3}{4} =$ • $\frac{-272}{8} =$ • (3.5)(-6.4) =
7.1.D Define and determine the absolute value of a number.	Students define absolute value as the distance of the number from zero. Examples: • Explain why 5 and -5 have the same absolute value. • Evaluate $\lvert 7.8 - 10.3 \rvert$.
7.1.E Solve two-step linear equations.	Example: • Solve $3.5x - 12 = 408$ and show each step in the process.
7.1.F Write an equation that corresponds to a given problem situation, and describe a problem situation that corresponds to a given equation.	Students have represented various types of problems with expressions and particular types of equations in previous grades. Many students at this grade level will also be able to deal with inequalities. Examples: • Meagan spent $56.50 on 3 blouses and a pair of jeans. If each blouse cost the same amount and the jeans cost $25, write an algebraic equation that represents this situation and helps you determine how much one blouse cost. • Describe a problem situation that could be solved using the equation $15 = 2x - 7$.

Performance Expectations

Students are expected to:

7.1.G Solve single- and multi-step word problems involving rational numbers and verify the solutions.

Explanatory Comments and Examples

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Example:

- Tom wants to buy some candy bars and magazines for a trip. He has decided to buy three times as many candy bars as magazines. Each candy bar costs \$0.70 and each magazine costs \$2.50. The sales tax rate on both types of items is $6\frac{1}{2}\%$. How many of each item can he buy if he has \$20.00 to spend?

Grade 7

***7.2. Core Content:** Proportionality and similarity* *(Operations, Geometry/Measurement, Algebra)*

Students extend their work with ratios to solve problems involving a variety of proportional relationships, such as making conversions between measurement units or finding the percent increase or decrease of an amount. They also solve problems involving the proportional relationships found in similar figures, and in so doing reinforce an important connection between numerical operations and geometric relationships. Students graph proportional relationships and identify the rate of change as the slope of the related line. The skills and concepts related to proportionality represent some of the most important connecting ideas across K–12 mathematics. With a good understanding of how things grow proportionally, students can understand the linear relationships that are the basis for much of high school mathematics. If learned well, proportionality can open the door for success in much of secondary mathematics.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
7.2.A Mentally add, subtract, multiply, and divide simple fractions, decimals, and percents.	Example: • A shirt is on sale for 20% off the original price of \$15. Use mental math strategies to calculate the sale price of the shirt.
7.2.B Solve single- and multi-step problems involving proportional relationships and verify the solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations. Problems include those that involve rate, percent increase or decrease, discount, markup, profit, interest, tax, or the conversion of money or measurement (including multiplying or dividing amounts in recipes). More complex problems, such as dividing 100 into more than two proportional parts (e.g., 4:3:3), allow students to generalize what they know about proportional relationships to a range of situations. Examples: • At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs and is going to give all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Explain your answer. • After eating at a restaurant, you know that the bill before tax is \$52.60 and that the sales tax rate is 8%. You decide to leave a 20% tip for the waiter based on the pre-tax amount. How much should you leave for the waiter? How much will the total bill be, including tax and tip? Show work to support your answers.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
7.2.B cont.	• Joe, Sam, and Jim completed different amounts of yard work around the school. They agree to split the $200 they earned in a ratio of 5:3:2, respectively. How much did each boy receive?
7.2.C Describe proportional relationships in similar figures and solve problems involving similar figures.	Students should recognize the constant ratios in similar figures and be able to describe the role of a scale factor in situations involving similar figures. They should be able to connect this work with more general notions of proportionality. Example: • The length of the shadow of a tree is 68 feet at the same time that the length of the shadow of a 6-foot vertical pole is 8 feet. What is the height of the tree?
7.2.D Make scale drawings and solve problems related to scale.	Example: • On an 80:1 scale drawing of the floor plan of a house, the dimensions of the living room are $1\frac{7}{8}'' \times 2\frac{1}{2}''$. What is the actual area of the living room in square feet?
7.2.E Represent proportional relationships using graphs, tables, and equations, and make connections among the representations.	Proportional relationships are linear relationships whose graphs pass through the origin and can be written in the form $y = kx$. Example: • The relationship between the width and length of similar rectangles is shown in the table below. Write an equation that expresses the length, *l*, in terms of the width, *w*, and graph the relationship between the two variables.

width	4	12	18	...	w
length	10	30	45	...	?

7.2.F Determine the slope of a line corresponding to the graph of a proportional relationship and relate slope to similar triangles.	This expectation connects the constant rate of change in a proportional relationship to the concept of slope of a line. Students should know that the slope of a line is the same everywhere on the line and realize that similar triangles can be used to demonstrate this fact. They should recognize how proportionality is reflected in slope as it is with similar triangles. A more complete discussion of slope is developed in high school.

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*

7.2.G Determine the unit rate in a proportional relationship and relate it to the slope of the associated line.

The associated unit rate, constant rate of change of the function, and slope of the graph all represent the constant of proportionality in a proportional relationship.

Example:

- Coffee costs $18.96 for 3 pounds. What is the cost per pound of coffee? Draw a graph of the proportional relationship between the number of pounds of coffee and the total cost, and describe how the unit rate is represented on the graph.

7.2.H Determine whether or not a relationship is proportional and explain your reasoning.

A proportional relationship is one in which two quantities are related by a constant scale factor, *k*. It can be written in the form $y = kx$. A proportional relationship has a constant rate of change and its graph passes through the origin.

Example:

- Determine whether each situation represents a proportional relationship and explain your reasoning.

—

x	1	2	3	4
y	4.5	9	13.5	18

— $y = 3x + 2$

— One way to calculate a person's maximum target heart rate during exercise in beats per minute is to subtract the person's age from 200. Is the relationship between the maximum target heart rate and age proportional? Explain your reasoning.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
7.2.I Solve single- and multi-step problems involving conversions within or between measurement systems and verify the solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations. Students should be given the conversion factor when converting between measurement systems. Examples: • The lot that Dana is buying for her new one-story house is 35 yards by 50 yards. Dana's house plans show that her house will cover 1,600 square feet of land. What percent of Dana's lot will not be covered by the house? Explain your work. • Joe was planning a business trip to Canada, so he went to the bank to exchange $200 U.S. dollars for Canadian dollars (at a rate of $1.02 CDN per $1 US). On the way home from the bank, Joe's boss called to say that the destination of the trip had changed to Mexico City. Joe went back to the bank to exchange his Canadian dollars for Mexican pesos (at a rate of 10.8 pesos per $1 CDN). How many Mexican pesos did Joe get?

Grade 7

***7.3. Core Content:** Surface area and volume* *(Algebra, Geometry/Measurement)*

Students extend their understanding of surface area and volume to include finding surface area and volume of cylinders and volume of cones and pyramids. They apply formulas and solve a range of problems involving three-dimensional objects, including problems people encounter in everyday life, in certain types of work, and in other school subjects. With a strong understanding of how to work with both two-dimensional and three-dimensional figures, students build an important foundation for the geometry they will study in high school.

Performance Expectations *Students are expected to:*	Explanatory Comments and Examples
7.3.A Determine the surface area and volume of cylinders using the appropriate formulas and explain why the formulas work.	Explanations might include the use of models such as physical objects or drawings. A net can be used to illustrate the formula for finding the surface area of a cylinder.
7.3.B Determine the volume of pyramids and cones using formulas.	
7.3.C Describe the effect that a change in scale factor on one attribute of a two- or three-dimensional figure has on other attributes of the figure, such as the side or edge length, perimeter, area, surface area, or volume of a geometric figure.	Examples: • A cube has a side length of 2 cm. If each side length is tripled, what happens to the surface area? What happens to the volume? • What happens to the area of a circle if the diameter is decreased by a factor of 3?
7.3.D Solve single- and multi-step word problems involving surface area or volume and verify the solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations. Examples: • Alexis needs to paint the four exterior walls of a large rectangular barn. The length of the barn is 80 feet, the width is 50 feet, and the height is 30 feet. The paint costs $28 per gallon, and each gallon covers 420 square feet. How much will it cost Alexis to paint the barn? Explain your work. • Tyesha has decided to build a solid concrete pyramid on her empty lot. The base will be a square that is forty feet by forty feet and the height will be thirty feet. The concrete that she will use to construct the pyramid costs $70 per cubic yard. How much will the concrete for the pyramid cost Tyesha? Justify your answer.

Grade 7

***7.4. Core Content:** Probability and data* *(Data/Statistics/Probability)*

Students apply their understanding of rational numbers and proportionality to concepts of probability. They begin to understand how probability is determined, and they make related predictions. Students revisit how to interpret data, now using more sophisticated types of data graphs and thinking about the meaning of certain statistical measures. Statistics, including probability, is considered one of the most important and practical fields of study for making sense of quantitative information, and it plays an important part in secondary mathematics in the 21st century.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
7.4.A Represent the sample space of probability experiments in multiple ways, including tree diagrams and organized lists.	The sample space is the set of all possible outcomes. Example: • José flips a penny, Jane flips a nickel, and Janice flips a dime, all at the same time. List the possible outcomes of the three simultaneous coin flips using a tree diagram or organized list.
7.4.B Determine the theoretical probability of a particular event and use theoretical probability to predict experimental outcomes.	Example: • A triangle with a base of 8 units and a height of 7 units is drawn inside a rectangle with an area of 90 square units. What is the probability that a randomly selected point inside the rectangle will also be inside the triangle? • There are 5 blue, 4 green, 8 red, and 3 yellow marbles in a paper bag. Rosa runs an experiment in which she draws a marble from the bag, notes the color on a sheet of paper, and puts the marble back in the bag, repeating the process 200 times. About how many times would you expect Rosa to draw a red marble?
7.4.C Describe a data set using measures of center (median, mean, and mode) and variability (maximum, minimum, and range) and evaluate the suitability and limitations of using each measure for different situations.	As a way to understand these ideas, students could construct data sets for a given mean, median, mode, or range. Examples: • Kiley keeps track of the money she spends each week for two months and records the following amounts: \$6.30, \$2.25, \$43.00, \$2.25, \$11.75, \$5.25, \$4.00, and \$5.20. Which measure of center is most representative of Kiley's weekly spending? Support your answer. • Construct a data set with five data points, a mean of 24, a range of 10, and without a mode. • A group of seven adults have an average age of 36. If the ages of three of the adults are 45, 30, and 42, determine possible ages for the remaining four adults.

Performance Expectations

Students are expected to:

7.4.D Construct and interpret histograms, stem-and-leaf plots, and circle graphs.

7.4.E Evaluate different displays of the same data for effectiveness and bias, and explain reasoning.

Explanatory Comments and Examples

Example:

- The following two bar graphs of the same data show the number of five different types of sodas that were sold at Blake High School. Compare and contrast the two graphs. Describe a reason why you might choose to use one graph over the other.

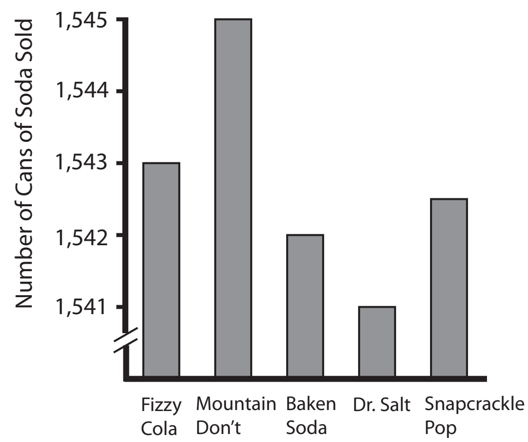


Figure 1

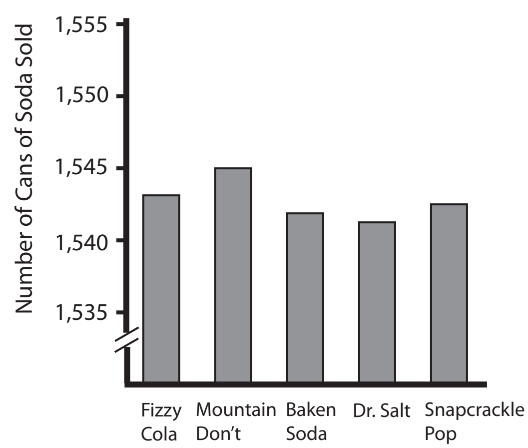


Figure 2

Grade 7

7.5. Additional Key Content *(Numbers, Algebra)*

Students extend their coordinate graphing skills to plotting points with both positive and negative coordinates on the coordinate plane. Using pairs of numbers to locate points is a necessary skill for reading maps and tables and a critical foundation for high school mathematics. Students further prepare for algebra by learning how to use exponents to write numbers in terms of their most basic (prime) factors.

Performance Expectations		***Explanatory Comments and Examples***
Students are expected to:		
7.5.A	Graph ordered pairs of rational numbers and determine the coordinates of a given point in the coordinate plane.	Example: • Graph and label the points A(1, 2), B(-1, 5), C(-3, 2), and D(-1, -5). Connect the points in the order listed and identify the figure formed by the four points. • Graph and label the points A(1, -2), B(-4, -2), and C(-4, 3). Determine the coordinates of the fourth point (D) that will complete the figure to form a square. Graph and label point D on the coordinate plane and draw the resulting square.
7.5.B	Write the prime factorization of whole numbers greater than 1, using exponents when appropriate.	Writing numbers in prime factorization is a useful tool for determining the greatest common factor and least common multiple of two or more numbers. Example: • Write the prime factorization of 360 using exponents.

Grade 7

***7.6. Core Processes:** Reasoning, problem solving, and communication*

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher-level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade seven, students solve problems that involve positive and negative numbers and often involve proportional relationships. As students solve these types of problems, they build a strong foundation for the study of linear functions that will come in grade eight.

Performance Expectations

Students are expected to:

7.6.A Analyze a problem situation to determine the question(s) to be answered.

7.6.B Identify relevant, missing, and extraneous information related to the solution to a problem.

7.6.C Analyze and compare mathematical strategies for solving problems, and select and use one or more strategies to solve a problem.

7.6.D Represent a problem situation, describe the process used to solve the problem, and verify the reasonableness of the solution.

7.6.E Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.

7.6.F Apply a previously used problem-solving strategy in a new context.

7.6.G Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.

7.6.H Make and test conjectures based on data (or information) collected from explorations and experiments.

Explanatory Comments and Examples

Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, physical objects, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Examples:

- When working on a report for class, Catrina read that a person over the age of 30 can lose approximately 0.06 centimeters of height per year. Catrina's 80-year-old grandfather is 5 feet 7 inches tall. Assuming her grandfather's height has decreased at this rate, about how tall was he at age 30? Catrina's cousin, Richard, is 30 years old and is 6 feet 3 inches tall. Assuming his height also decreases approximately 0.06 centimeters per year after the age of 30, about how tall will you expect him to be at age 55? (Remember that 1 inch $\approx$ 2.54 centimeters.) Justify your solution.
- If one man takes 1.5 hours to dig a 5-ft $\times$ 5-ft $\times$ 3-ft hole, how long will it take three men working at the same pace to dig a 10-ft $\times$ 12-ft $\times$ 3-ft hole? Explain your solution.

GRADE 8
STANDARDS

Grade 8

8.1. Core Content: *Linear functions and equations* *(Algebra)*

Students solve a variety of linear equations and inequalities. They build on their familiarity with proportional relationships and simple linear equations to work with a broader set of linear relationships, and they learn what functions are. They model applied problems with mathematical functions represented by graphs and other algebraic techniques. This Core Content area includes topics typically addressed in a high school algebra or a first-year integrated math course, but here this content is expected of all middle school students in preparation for a rich high school mathematics program that goes well beyond these basic algebraic ideas.

Performance Expectations *Students are expected to:*		***Explanatory Comments and Examples***
8.1.A	Solve one-variable linear equations.	Examples: Solve each equation for x. • $91 - 2.5x = 26$ • $\frac{7}{8}(x-2)=119$ • $-3x + 34 = 5x$ • $114 = -2x - 8 + 5x$ • $3(x - 2) - 4x = 2(x + 22) - 5$
8.1.B	Solve one- and two-step linear inequalities and graph the solutions on the number line.	The emphasis at this grade level is on gaining experience with inequalities, rather than on becoming proficient at solving inequalities in which multiplying or dividing by a negative is necessary. Example: • Graph the solution of $4x - 21 > 57$ on the number line.
8.1.C	Represent a linear function with a verbal description, table, graph, or symbolic expression, and make connections among these representations.	Translating among these various representations of functions is an important way to demonstrate a conceptual understanding of functions.
8.1.D	Determine the slope and *y*-intercept of a linear function described by a symbolic expression, table, or graph.	Examples: • Determine the slope and *y*-intercept for the function described by $y=\frac{2}{3}x-5$ • The following table represents a linear function. Determine the slope and *y*-intercept.

x	2	3	5	8	12
y	5	8	14	23	35

Performance Expectations *Students are expected to:*	Explanatory Comments and Examples
8.1.E Interpret the slope and y-intercept of the graph of a linear function representing a contextual situation.	Example: • A car is traveling down a long, steep hill. The elevation, E, above sea level (in feet) of the car when it is d miles from the top of the hill is given by $E = 7500 - 250d$, where d can be any number from 0 to 6. Find the slope and y-intercept of the graph of this function and explain what they mean in the context of the moving car.
8.1.F Solve single- and multi-step word problems involving linear functions and verify the solutions.	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations. Example: • Mike and Tim leave their houses at the same time to walk to school. Mike's walk can be represented by $d_1 = 4000 - 400t$, and Tim's walk can be represented by $d_2 = 3400 - 250t$, where d is the distance from the school in feet and t is the walking time in minutes. Who arrives at school first? By how many minutes? Is there a time when Mike and Tim are the same distance away from the school? Explain your reasoning.
8.1.G Determine and justify whether a given verbal description, table, graph, or symbolic expression represents a linear relationship.	Examples: • Could the data presented in the table represent a linear function? Explain your reasoning.

x	-1	0	1	2	3	4	5
y	0	-1	0	3	8	15	24

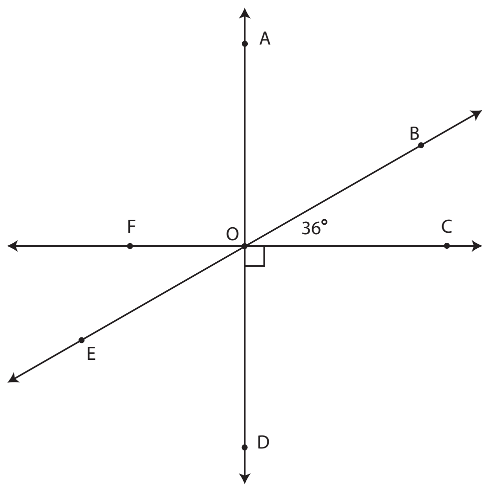
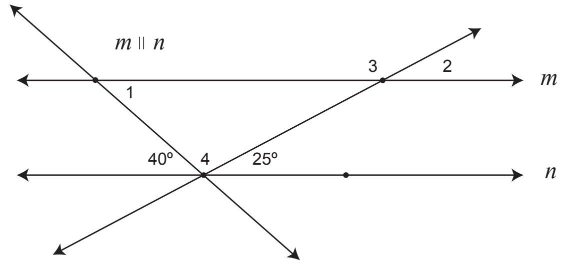
• Does $y = \frac{1}{4}x - 5$ represent a linear function?

Explain your reasoning.

Grade 8

***8.2. Core Content:** Properties of geometric figures* *(Numbers, Geometry/Measurement)*

Students work with lines and angles, especially as they solve problems involving triangles. They use known relationships involving sides and angles of triangles to find unknown measures, connecting geometry and measurement in practical ways that will be useful well after high school. Since squares of numbers arise when using the Pythagorean Theorem, students work with squares and square roots, especially in problems with two- and three-dimensional figures. Using basic geometric theorems such as the Pythagorean Theorem, students get a preview of how geometric theorems are developed and applied in more formal settings, which they will further study in high school.

Performance Expectations *Students are expected to:*	Explanatory Comments and Examples
8.2.A Identify pairs of angles as complementary, supplementary, adjacent, or vertical, and use these relationships to determine missing angle measures.	Example: • Determine the measures of ∠BOA, ∠EOD, ∠FOB, and ∠FOE and explain how you found each measure. As part of your explanation, identify pairs of angles as complementary, supplementary, or vertical. 
8.2.B Determine missing angle measures using the relationships among the angles formed by parallel lines and transversals.	Example: • Determine the measures of the indicated angles. ∠1: _____ ∠2: _____ ∠3: _____ ∠4: _____ 

Performance Expectations *Students are expected to:*		*Explanatory Comments and Examples*
8.2.C	Demonstrate that the sum of the angle measures in a triangle is 180 degrees, and apply this fact to determine the sum of the angle measures of polygons and to determine unknown angle measures.	Examples: • Determine the measure of each interior angle in a regular pentagon. • In a certain triangle, the measure of one angle is four times the measure of the smallest angle, and the measure of the remaining angle is the sum of the measures of the other two angles. Determine the measure of each angle.
8.2.D	Represent and explain the effect of one or more translations, rotations, reflections, or dilations (centered at the origin) of a geometric figure on the coordinate plane.	Example: • Consider a trapezoid with vertices (1, 2), (1, 6), (6, 4), and (6, 2). The trapezoid is reflected across the *x*-axis and then translated four units to the left. Graph the image of the trapezoid after these two transformations and give the coordinates of the new vertices.
8.2.E	Quickly recall the square roots of the perfect squares from 1 through 225 and estimate the square roots of other positive numbers.	Students can use perfect squares of integers to determine squares and square roots of related numbers, such as 1.96 and 0.0049. Examples: • Determine: $\sqrt{36}$, $\sqrt{0.25}$, $\sqrt{144}$, and $\sqrt{196}$. • Between which two consecutive integers does the square root of 74 lie?
8.2.F	Demonstrate the Pythagorean Theorem and its converse and apply them to solve problems.	One possible demonstration is to start with a right triangle, use each of the three triangle sides to form the side of a square, and draw the remaining three sides of each of the three squares. The areas of the three squares represent the Pythagorean relationship. Examples: • Is a triangle with side lengths 5 cm, 12 cm, and 13 cm a right triangle? Why or why not? • Determine the length of the diagonal of a rectangle that is 7 ft by 10 ft.
8.2.G	Apply the Pythagorean Theorem to determine the distance between two points on the coordinate plane.	Example: • Determine the distance between the points (⁻2, 3) and (4, 7).

Grade 8

***8.3. Core Content:** Summary and analysis of data sets* *(Algebra, Data/Statistics/Probability)*

Students build on their extensive experience organizing and interpreting data and apply statistical principles to analyze statistical studies or short statistical statements, such as those they might encounter in newspapers, on television, or on the Internet. They use mean, median, and mode to summarize and describe information, even when these measures may not be whole numbers. Students use their knowledge of linear functions to analyze trends in displays of data. They create displays for two sets of data in order to compare the two sets and draw conclusions. They expand their work with probability to deal with more complex situations than they have previously seen. These concepts of statistics and probability are important not only in students' lives, but also throughout the high school mathematics program.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
8.3.A Summarize and compare data sets in terms of variability and measures of center.	Students use mean, median, mode, range, and interquartile range to summarize and compare data sets, and explain the influence of outliers on each measure. Example: • Captain Bob owns two charter boats, the *Sock-Eye-To-Me* and *Old Gus*, which take tourists on fishing trips. On Saturday, the *Sock-Eye-To-Me* took four people fishing and returned with eight fish weighing 18, 23, 20, 6, 20, 22, 18, and 20 pounds. On the same day, *Old Gus* took five people fishing and returned with ten fish weighing 38, 18, 12, 14, 17, 42, 12, 16, 12, and 14 pounds. Using measures of center and variability, compare the weights of the fish caught by the people in the two boats. Make a summary statement telling which boat you would charter for fishing based on these data and why. What influence, if any, do outliers have on the particular statistics for these data?
8.3.B Select, construct, and analyze data displays, including box-and-whisker plots, to compare two sets of data.	Previously studied displays include stem-and-leaf plots, histograms, circle graphs, and line plots. Here these displays are used to compare data sets. Box-and-whisker plots are introduced here for the first time as a powerful tool for comparing two or more data sets.

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*

8.3.B cont.

Example:

- As part of their band class, Tayla and Alyssa are required to keep practice records that show the number of minutes they practice their instruments each day. Below are their practice records for the past fourteen days:

  Tayla: 55, 45, 60, 45, 30, 30, 90, 50, 40, 75, 25, 90, 105, 60

  Alyssa: 20, 120, 25, 20, 0, 15, 30, 15, 90, 0, 30, 30, 10, 30

  Of stem-and-leaf plot, circle graph, or line plot, select the data display that you think will best compare the two girls' practice records. Construct a display to show the data. Compare the amount of time the two girls practice by analyzing the data presented in the display.

8.3.C Create a scatterplot for a two-variable data set, and, when appropriate, sketch and use a trend line to make predictions.

Example:

- Kera randomly selected seventeen students from her middle school for a study comparing arm span to standing height. The students' measurements are shown in the table below.

  Comparison of Arm Span and Standing Height (in cm) at Icicle River Middle School

Height (cm)	Arm Span (cm)	Height (cm)	Arm Span (cm)
138	145	155	150
135	135	175	177
142	147	162	160
158	145	150	152
177	174	142	143
150	152	149	149
158	162	160	165
160	160	173	170
160	158		

  Create a scatterplot for the data shown.

  If appropriate, sketch a trendline.

  Use these data to estimate the arm span of a student with a height of 180 cm, and the height of a student with an arm span of 130 cm. Explain any limitations of using this process to make estimates.

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
8.3.D Describe different methods of selecting statistical samples and analyze the strengths and weaknesses of each method.	Students should work with a variety of sampling techniques and should be able to identify strengths and weakness of random, census, convenience, and representative sampling. Example: • Carli, Jamar, and Amberly are conducting a survey to determine their school's favorite Seattle professional sports team. Carli selects her sample using a convenience method—she surveys students on her bus during the ride to school. Jamar uses a computer to randomly select 30 numbers from 1 through 600, and then surveys the corresponding students from a numbered, alphabetical listing of the student body. Amberly waits at the front entrance before school and surveys every twentieth student entering. Analyze the strengths and weaknesses of each method.
8.3.E Determine whether conclusions of statistical studies reported in the media are reasonable.	
8.3.F Determine probabilities for mutually exclusive, dependent, and independent events for small sample spaces.	Examples: • Given a standard deck of 52 playing cards, what is the probability of drawing a king or queen? [Some students may be unfamiliar with playing cards, so alternate examples may be desirable.] • Leyanne is playing a game at a birthday party. Beneath ten paper cups, a total of five pieces of candy are hidden, with one piece hidden beneath each of five cups. Given only three guesses, Leyanne must uncover three pieces of candy to win all the hidden candy. What is the probability she will win all the candy? • A bag contains 7 red marbles, 5 blue marbles, and 8 green marbles. If one marble is drawn at random and put back in the bag, and then a second marble is drawn at random, what is the probability of drawing first a red marble, then a blue marble?

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	
8.3.G Solve single- and multi-step problems using counting techniques and Venn diagrams and verify the solutions	The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations. Counting techniques include the fundamental counting principle, lists, tables, tree diagrams, etc. Examples: • Jack's Deli makes sandwiches that include a choice of one type of bread, one type of cheese, and one type of meat. How many different sandwiches could be made given 4 different bread types, 3 different cheeses, and 5 different meats? Explain your reasoning. • A small high school has 57 tenth-graders. Of these students, 28 are taking geometry, 34 are taking biology, and 10 are taking neither geometry nor biology. How many students are taking both geometry and biology? How many students are taking geometry but not biology? How many students are taking biology but not geometry? Draw a Venn diagram to illustrate this situation.

Grade 8

8.4. Additional Key Content *(Numbers, Operations)*

Students deal with a few key topics about numbers as they prepare to shift to higher level mathematics in high school. First, they use scientific notation to represent very large and very small numbers, especially as these numbers are used in technological fields and in everyday tools like calculators or personal computers. Scientific notation has become especially important as “extreme units” continue to be identified to represent increasingly tiny or immense measures arising in technological fields. A second important numerical skill involves using exponents in expressions containing both numbers and variables. Developing this skill extends students’ work with order of operations to include more complicated expressions they might encounter in high school mathematics. Finally, to help students understand the full breadth of the real-number system, students are introduced to simple irrational numbers, thus preparing them to study higher level mathematics in which properties and procedures are generalized for the entire set of real numbers.

Performance Expectations *Students are expected to:*		***Explanatory Comments and Examples***
8.4.A	Represent numbers in scientific notation, and translate numbers written in scientific notation into standard form.	Examples: • Represent 4.27×10^{-3} in standard form. • Represent 18,300,000 in scientific notation. • Throughout the year 2004, people in the city of Cantonville sent an average of 400 million text messages a day. Using this information, about how many total text messages did Cantonville residents send in 2004? (2004 was a leap year.) Express your answer in scientific notation.
8.4.B	Solve problems involving operations with numbers in scientific notation and verify solutions.	Units include those associated with technology, such as nanoseconds, gigahertz, kilobytes, teraflops, etc. Examples: • A supercomputer used by a government agency will be upgraded to perform 256 teraflops (that is, 256 trillion calculations per second). Before the upgrade, the supercomputer performs 1.6×10^{13} calculations per second. How many more calculations per second will the upgraded supercomputer be able to perform? Express the answer in scientific notation. • A nanosecond is one billionth of a second. How many nanoseconds are there in five minutes? Express the answer in scientific notation.

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
8.4.C Evaluate numerical expressions involving non-negative integer exponents using the laws of exponents and the order of operations.	Example: • Simplify and write the answer in exponential form: $\frac{(7^4)^2}{7^3}$ Some students will be ready to solve problems involving simple negative exponents and should be given the opportunity to do so. Example: • Simplify and write the answer in exponential form: $(5^4)^2 5^{-3}$
8.4.D Identify rational and irrational numbers.	Students should know that rational numbers are numbers that can be represented as the ratio of two integers; that the decimal expansions of rational numbers have repeating patterns, or *terminate*; and that there are numbers that are not rational. Example: • Identify whether each number is rational or irrational and explain your choice. $3.14,\ 4.\overline{6},\ \frac{1}{11},\ \sqrt{2},\ \sqrt{25},\ \pi$

Grade 8

***8.5. Core Processes:** Reasoning, problem solving, and communication*

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade eight, students solve problems that involve proportional relationships and linear relationships, including applications found in many contexts outside of school. These problems dealing with proportionality continue to be important in many applied contexts, and they lead directly to the study of algebra. Students also begin to deal with informal proofs for theorems that will be proven more formally in high school.

Performance Expectations

Students are expected to:

8.5.A Analyze a problem situation to determine the question(s) to be answered.

8.5.B Identify relevant, missing, and extraneous information related to the solution to a problem.

8.5.C Analyze and compare mathematical strategies for solving problems, and select and use one or more strategies to solve a problem.

8.5.D Represent a problem situation, describe the process used to solve the problem, and verify the reasonableness of the solution.

8.5.E Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.

8.5.F Apply a previously used problem-solving strategy in a new context.

8.5.G Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.

8.5.H Make and test conjectures based on data (or information) collected from explorations and experiments.

Explanatory Comments and Examples

Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Examples:

- The dimensions of a room are 12 feet by 15 feet by 10 feet. What is the furthest distance between any two points in the room? Explain your solution.
- Miranda's phone service contract ends this month. She is looking for ways to save money and is considering changing phone companies. Her current cell phone carrier, X-Cell, calculates the monthly bill using the equation $c = \$15.00 + \$0.07m$, where c represents the total monthly cost and m represents the number of minutes of talk time during a monthly billing cycle. Another company, Prism Cell, offers 300 free minutes of talk time each month for a base fee of $30.00 with an additional $0.15 for every minute over 300 minutes. Miranda's last five phone bills were $34.95, $36.70, $37.82, $62.18, and $36.28. Using the data from the last five months, help Miranda decide whether she should switch companies. Justify your answer.

MATHEMATICS 1
STANDARDS

Mathematics 1

In Mathematics 1, students begin to formalize mathematics by exploring function concepts with emphasis on the family of linear functions and their applications. Students extend their work with graphical and numerical data analysis to include bivariate data involving linear relationships. Students identify and prove relationships about lines in the plane and similar triangles. Proportionality is a common thread in Mathematics 1 that connects linear functions, data analysis, and coordinate geometry. Throughout this course, students develop their reasoning skills by making conjectures and predictions or creating simple proofs related to algebraic, geometric, and statistical relationships.

Mathematics 1

M1.1. Core Content: *Solving problems* *(Algebra)*

Students learn to solve many new types of problems in Mathematics 1, and this first core content area highlights the types of problems students will be able to solve after they master the concepts and skills in this course. Throughout Mathematics 1, students spend considerable time with linear functions and are introduced to other types of functions, including exponential functions and functions defined piecewise. They learn that specific functions model situations described in word problems, and thus they learn the broader notion that functions are used to solve various types of problems. The ability to write an equation that represents a problem is an important mathematical skill in itself, and each new function provides students the tool to solve yet another class of problems. Many problems that initially appear to be very different from each other can actually be represented by identical equations. This is an important and unifying principle of algebra—that the same algebraic techniques can be applied to a wide variety of different situations.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M1.1.A Select and justify functions and equations to model and solve problems.	Students can analyze the rate of change of a function represented with a table or graph to determine if the function is linear. Students also analyze common ratios to determine if the function is exponential. After selecting a function to model a situation, students describe appropriate domain restrictions. They use the function to solve the problem and interpret the solution in the context of the original situation. Examples: • A cup is 6 cm tall, including a 1.1 cm lip. Find a function that represents the height of a stack of cups in terms of the number of cups in the stack. Find a function that represents the number of cups in a stack of a given height. • For the month of July, Michelle will be dog-sitting for her very wealthy, but eccentric, neighbor, Mrs. Buffett. Mrs. Buffett offers Michelle two different salary plans: Plan 1: \$100 per day for the 31 days of the month. Plan 2: \$1 for July 1, \$2 for July 2, \$4 for July 3, and so on, with the daily rate doubling each day. a. Write functions that model the amount of money Michelle will earn each day on Plan 1 and Plan 2. Justify the functions you wrote. b. State an appropriate domain for each of the models based on the context. c. Which plan should Michelle choose to maximize her earnings? Justify your recommendation mathematically. d. Extension: Write an algebraic function for the cumulative pay for each plan based on the number of days worked.

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
M1.1.B Solve problems that can be represented by linear functions, equations, and inequalities.	It is mathematically important to represent a word problem as an equation. Students must analyze the situation and find a way to represent it mathematically. After solving the equation, students think about the solution in terms of the original problem. Examples: • The assistant pizza maker makes 6 pizzas an hour. The master pizza maker makes 10 pizzas an hour but starts baking two hours later than his assistant. Together, they must make 92 pizzas. How many hours from when the assistant starts baking will it take? What is a general equation, in function form, that could be used to determine the number of pizzas that can be made in two or more hours? • A swimming pool holds 375,000 liters of water. Two large hoses are used to fill the pool. The first hose fills at the rate of 1,500 liters per hour and the second hose fills at the rate of 2,000 liters per hour. How many hours does it take to fill the pool completely?
M1.1.C Solve problems that can be represented by a system of two linear equations or inequalities.	Examples: • An airplane flies from Baltimore to Seattle (assume a distance of 2,400 miles) in 7 hours, but the return flight takes only $4\frac{1}{4}$ hours. The air speed of the plane is the same in both directions. How many miles per hour does the plane fly with respect to the wind? What is the wind speed in miles per hour? • A coffee shop employee has one cup of 85% milk (the rest is chocolate) and another cup of 60% milk (the rest is chocolate). He wants to make one cup of 70% milk How much of the 85% milk and 60% milk should he mix together to make the 70% milk? • Two plumbing companies charge different rates for their service. Clyde's Plumbing Company charges a \$75-per-visit fee that includes one hour of labor plus \$45 dollars per hour after the first hour. We-Unclog-It Plumbers charges a \$100-per-visit fee that includes one hour of labor plus \$40 per hour after the first hour. For how many hours of plumbing work would Clyde's be less expensive than We-Unclog-It? Note: Although this context is discrete, students can model it with continuous linear functions.

Performance Expectations

Students are expected to:

M1.1.D Solve problems that can be represented by exponential functions and equations.

Explanatory Comments and Examples

Students recognize common examples of exponential growth or decay, such as applying exponential functions to determine compound interest, population growth, and radioactivity. They approximate solutions with graphs or tables, check solutions numerically, and when possible, solve problems exactly.

Example:

- Mr. Tsu invests $1000 in a 5-year CD that pays 4% interest compounded yearly. Present to Mr. Tsu his expected balance at the end of years 1, 3, and 5 and the process you used to arrive at each value.

Mathematics 1

***M1.2. Core Content:** Characteristics and behaviors of functions* *(Algebra)*

Students formalize and deepen their understanding of functions, the defining characteristics and uses of functions, and the mathematical language used to describe functions. They learn that functions are often specified by an equation of the form $y = f(x)$, where any allowable x-value yields a unique y-value. Mathematics 1 has a particular focus on linear functions, equations, and systems of equations and on functions that can be defined piecewise, particularly step functions and functions that contain the absolute value of an expression. Students compare and contrast non-linear functions, such as quadratic and exponential, with linear functions. They learn about the representations and basic transformations of these functions and the practical and mathematical limitations that must be considered when working with functions and when using functions to model situations.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***

M1.2.A Determine whether a relationship is a function and identify the domain, range, roots, and independent and dependent variables.	Functions studied in Mathematics 1 include linear and those defined piecewise (including step functions and those that contain the absolute value of an expression). They compare and contrast non-linear functions, such as quadratic and exponential, to linear functions.
	
	Given a problem situation, students should describe further restrictions on the domain of a function that are appropriate for the problem context.
	
	Examples:
	• Which of the following are functions? Explain why or why not.
	    — The age in years of each student in your math class and each student's shoe size.
	    — The number of degrees a person rotates a spigot and the volume of water that comes out of the spigot.
	• A function $f(n) = 60n$ is used to model the distance in miles traveled by a car traveling 60 miles per hour in n hours. Identify the domain and range of this function. What restrictions on the domain of this function should be considered for the model to correctly reflect the situation?
	• What is the domain of $f(x) = \sqrt{5 - x}$?

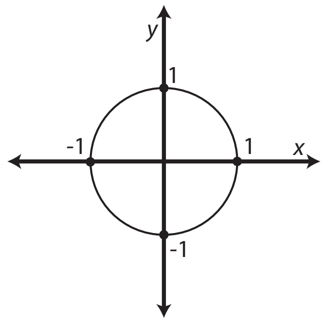
Performance Expectations

Students are expected to:

M1.2.A cont.

Explanatory Comments and Examples

- Which of the following equations, inequalities, or graphs determine y as a function of x?
  - $y = 2$
  - $x = 3$
  - $y = |x|$
  - $y = \begin{cases} x+3, & x \le 1 \\ x-2, & x > 1 \end{cases}$
  - $x^2 + y^2 = 1$



M1.2.B Represent a function with a symbolic expression, as a graph, in a table, and using words, and make connections among these representations.

This expectation applies each time a new class (family) of functions is encountered. In Mathematics 1, students should be introduced to a variety of additional functions that include expressions such as x^3, $\sqrt{x}$, $\frac{1}{x}$, and absolute values. They will study these functions in depth in subsequent courses.

Students should know that $f(x) = \frac{a}{x}$ represents an inverse variation. Students begin to describe the graph of a function from its symbolic expression, and use key characteristics of the graph of a function to infer properties of the related symbolic expression.

Translating among these various representations of functions is an important way to demonstrate conceptual understanding of functions.

Students learn that each representation has particular advantages and limitations. For example, a graph shows the shape of a function, but not exact values. They also learn that a table of values may not uniquely determine a single function without some specification of the nature of that function (e.g., it is quadratic).

Performance Expectations

Students are expected to:

M1.2.C Evaluate $f(x)$ at a (i.e., $f(a)$) and solve for x in the equation $f(x) = b$.

Explanatory Comments and Examples

Functions may be described and evaluated with symbolic expressions, tables, graphs, or verbal descriptions.

Students should distinguish between solving for $f(x)$ and evaluating a function at x.

Example:

- Roses-R-Red sells its roses for $0.75 per stem and charges a $20 delivery fee per order.
  - What is the cost of having 10 roses delivered?
  - How many roses can you have delivered for $65?

M1.2.D Plot points, sketch, and describe the graphs of functions of the form $f(x) = \frac{a}{x} + b$.

Mathematics 1 addresses only rational functions of the form $f(x) = \frac{a}{x} + b$. Rational functions of the form $f(x) = \frac{a}{x^2} + b$ and $f(x) = \frac{a}{(bx+c)}$ are addressed in Mathematics 3.

Example:

- Sketch the graphs of the four functions $f(x) = \frac{a}{x} + b$ when $a = 4$ and 8 and $b = 0$ and 1.

Mathematics 1

***M1.3. Core Content:** Linear functions, equations, and relationships* *(Algebra, Geometry/ Measurement, Data/Statistics/Probability)*

Students understand that linear functions can be used to model situations involving a constant rate of change. They build on the work done in middle school to solve systems of linear equations and inequalities in two variables, learning to interpret the intersection of lines as the solution. While the focus is on solving equations, students also learn graphical and numerical methods for approximating solutions to equations. They use linear functions to analyze relationships, represent and model problems, and answer questions. These algebraic skills are applied in other Core Content areas across high school courses.

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
M1.3.A Write and solve linear equations and inequalities in one variable.	This expectation includes the use of absolute values in the equations and inequalities. Examples: • Write an absolute value equation or inequality for all the numbers 2 units from 7, and all the numbers that are more than b units from a. • Solve $\|x - 6\| \le 4$ and locate the solution on the number line. • Write an equation or inequality that has no real solutions; infinite numbers of real solutions; and exactly one real solution. • Solve for x in $2(x - 3) + 4x = 15 + 2x$. • Solve $8.5 < 3x + 2 \le 9.7$ and locate the solution on the number line.
M1.3.B Describe how changes in the parameters of linear functions and functions containing an absolute value of a linear expression affect their graphs and the relationships they represent.	In the case of a linear function $y = f(x)$, expressed in slope-intercept form ($y = mx + b$), m and b are parameters. Students should know that $f(x) = kx$ represents a direct variation (proportional relationship). Examples: • Graph a function of the form $f(x) = kx$, describe the effect that changes on k have on the graph and on $f(x)$, and answer questions that arise in proportional situations. • A gas station's 10,000-gallon underground storage tank contains 1,000 gallons of gasoline. Tanker trucks pump gasoline into the tank at a rate of 400 gallons per minute. How long will it take to fill the tank? Find a function that represents this situation and then graph the function. If the flow rate increases from 400 to 500 gallons per minute, how will the graph of the function change? If the initial amount of gasoline in the tank changes from 1,000 to 2,000 gallons, how will the graph of the function change? • Compare and contrast the functions $y = 3\|x\|$ and $y = -\frac{1}{3}\|x\|$.

Performance Expectations

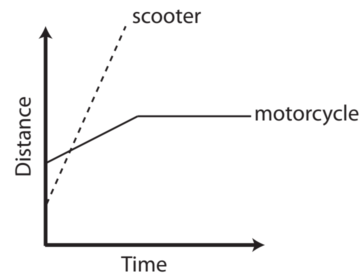
Students are expected to:

M1.3.C Identify and interpret the slope and intercepts of a linear function, including equations for parallel and perpendicular lines.

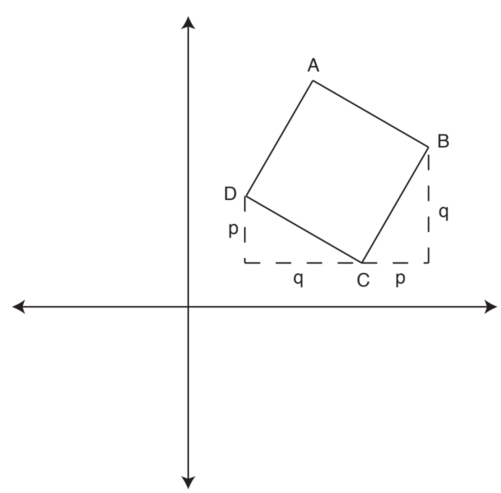
Explanatory Comments and Examples

Examples:

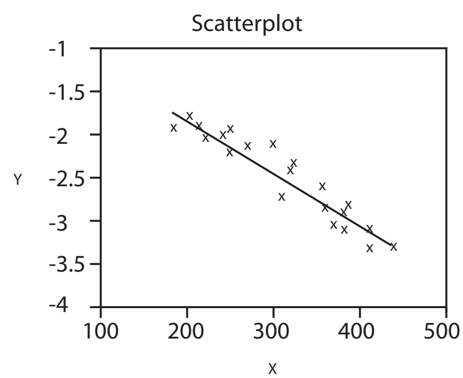
- The graph shows the relationship between time and distance from a gas station for a motorcycle and a scooter. What can be said about the relative speed of the motorcycle and scooter that matches the information in the graph? What can be said about the intersection of the graphs of the scooter and the motorcycle? Is it possible to tell which vehicle is further from the gas station at the initial starting point represented in the graph? At the end of the time represented in the graph? Why or why not?



- A 1,500-gallon tank contains 200 gallons of water. Water begins to run into the tank at the rate of 75 gallons per hour. When will the tank be full? Find a linear function that models this situation, draw a graph, and create a table of data points. Once you have answered the question and completed the tasks, explain your reasoning. Interpret the slope and *y*-intercept of the function in the context of the situation.
- Given that the figure below is a square, find the slope of the perpendicular sides AB and BC. Describe the relationship between the two slopes.



Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
M1.3.D Write and graph an equation for a line given the slope and the *y*-intercept, the slope and a point on the line, or two points on the line, and translate between forms of linear equations.	Linear equations may be written in slope-intercept, point-slope, and standard form. Examples: • Find an equation for a line with *y*-intercept equal to 2 and slope equal to 3. • Find an equation for a line with a slope of 2 that goes through the point (1, 1). • Find an equation for a line that goes through the points (-3, 5) and (6, -2). • For each of the following, use only the equation (without sketching the graph) to describe the graph. — $y = 2x + 3$ — $y – 7 = 2(x – 2)$ • Write the equation $3x + 2y = 5$ in slope intercept form. • Write the equation $y – 1 = 2(x – 2)$ in standard form.
M1.3.E Write and solve systems of two linear equations and inequalities in two variables.	Students solve both symbolic and word problems, understanding that the solution to a problem is given by the coordinates of the intersection of the two lines when the lines are graphed in the same coordinate plane. Examples: • Solve the following simultaneous linear equations algebraically: $-2x + y = 2$ $x + y = -1$ • Graph the above two linear equations on the same coordinate plane and use the graph to verify the algebraic solution. • An academic team is going to a state mathematics competition. There are 30 people going on the trip. There are 5 people who can drive and 2 types of vehicles, vans and cars. A van seats 8 people, and a car seats 4 people, including drivers. How many vans and cars does the team need for the trip? Explain your reasoning. Let *v* = number of vans and *c* = number of cars. $v + c \le 5$ $8v + 4c \ge 30$

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
M1.3.F Find the equation of a linear function that best fits bivariate data that are linearly related, interpret the slope and *y*-intercept of the line, and use the equation to make predictions.	A bivariate set of data presents data on two variables, such as shoe size and height. In high school, the emphasis is on using a line of best fit to interpret data and on students making judgments about whether a bivariate data set can be modeled with a linear function. Students can use various methods, including technology, to obtain a line of best fit. Making predictions involves both interpolating and extrapolating from the original data set. Students need to be able to evaluate the quality of their predictions, recognizing that extrapolation is based on the assumption that the trend indicated continues beyond the unknown data.
M1.3.G Describe the correlation of data in scatterplots in terms of strong or weak and positive or negative.	Example: • Which words—*strong* or *weak*, *positive* or *negative*—could be used to describe the correlation shown in the sample scatterplot below? 
M1.3.H Determine the equation of a line in the coordinate plane that is described geometrically, including a line through two given points, a line through a given point parallel to a given line, and a line through a given point perpendicular to a given line.	Examples: • Write an equation for the perpendicular bisector of a given line segment. • Determine the equation of a line through the points (5, 3) and (5, -2). • Prove that the slopes of perpendicular lines are negative inverses of each other.

Mathematics 1

***M1.4. Core Content:** Proportionality, similarity, and geometric reasoning* *(Geometry/Measurement)*

Students extend and formalize their knowledge of two-dimensional geometric figures and their properties, with a focus on properties of lines, angles, and triangles. They explain their reasoning using precise mathematical language and symbols. Students study basic properties of parallel and perpendicular lines, their respective slopes in the coordinate plane, and the properties of the angles formed when parallel lines are intersected by a transversal. They prove related theorems and apply them to solve problems that are purely mathematical and that arise in applied contexts. Students formalize their prior work with similarity and proportionality by making and proving conjectures about triangle similarity.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M1.4.A Distinguish between inductive and deductive reasoning.	Students generate and test conjectures inductively and then prove (or disprove) their conclusions deductively. Example: • A student first hypothesizes that the sum of the angles of a triangle is 180 degrees and then proves this is true. When was the student using inductive reasoning? When was s/he using deductive reasoning? Justify your answers.
M1.4.B Use inductive reasoning to make conjectures, to test the plausibility of a geometric statement, and to help find a counterexample.	Example: • Using dynamic geometry software, decide if the following is a plausible conjecture: If two parallel lines are cut by a transversal, then alternate interior angles are equal.
M1.4.C Use deductive reasoning to prove that a valid geometric statement is true.	Valid proofs may be presented in paragraph, two-column, or flow-chart formats. Proof by contradiction is a form of deductive reasoning. Example: • Prove that if two parallel lines are cut by a transversal, then alternate interior angles are equal.
M1.4.D Determine and prove triangle similarity.	Similarity in Mathematics 1 builds on proportionality concepts from middle school mathematics. Determining and proving triangle congruence and other properties of triangles are included in Mathematics 2. Students should identify necessary and sufficient conditions for similarity in triangles, and use these conditions in proofs. Example: • For a given ΔRST, prove that ΔXYZ, formed by joining the midpoints of the sides of ΔRST, is similar to ΔRST.

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
M1.4.E Know, prove, and apply theorems about parallel and perpendicular lines.	Students should be able to summarize and explain basic theorems. They are not expected to recite lists of theorems, but they should know the conclusion of a theorem when given its hypothesis. Examples: • Prove that a point on the perpendicular bisector of a line segment is equidistant from the ends of the line segment. • If each of two lines is perpendicular to a given line, what is the relationship between the two lines? How do you know?
M1.4.F Know, prove, and apply theorems about angles, including angles that arise from parallel lines intersected by a transversal.	Example: • Take two parallel lines *l* and *m*, with (distinct) points A and B on *l* and C and D on *m*. If $\overleftrightarrow{AC}$ intersects $\overleftrightarrow{BD}$ at point E, prove that $\Delta ABE \sim \Delta CDE$.
M1.4.G Explain and perform basic compass and straightedge constructions related to parallel and perpendicular lines.	Constructions using circles and lines with dynamic geometry software (i.e., virtual compass and straightedge) are equivalent to paper and pencil constructions. Example: • Construct and mathematically justify the steps to: — Bisect a line segment. — Drop a perpendicular from a point to a line. — Construct a line through a point that is parallel to another line.

Mathematics 1

M1.5. Core Content: *Data and distributions* *(Data/Statistics/Probability)*

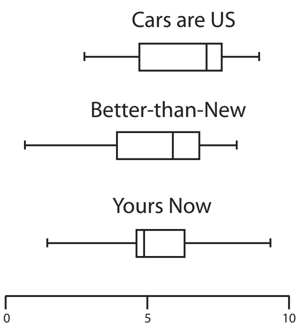
Students select mathematical models for data sets and use those models to represent, describe, and compare data sets. They analyze the linear relationship between two statistical variables and make and defend appropriate predictions, conjectures, and generalizations based on data. Students understand limitations of conclusions drawn from the results of a study or an experiment and recognize common misconceptions and misrepresentations.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M1.5.A Use and evaluate the accuracy of summary statistics to describe and compare data sets.	A univariate set of data identifies data on a single variable, such as shoe size.

This expectation extends what students have learned in earlier grades to include evaluation and justification. They both compute and evaluate the appropriateness of measure of center and spread (range and interquartile range) and use these measures to accurately compare data sets. Students will draw appropriate conclusions through the use of statistical measures of center, frequency, and spread, combined with graphical displays.

Examples:

- The local minor league baseball team has a salary dispute. Players claim they are being underpaid, but managers disagree.
  - — Bearing in mind that a few top players earn salaries that are quite high, would it be in the managers' best interest to use the mean or median when quoting the "average" salary of the team? Why?
  - — What would be in the players' best interest?
- Each box-and-whisker plot shows the prices of used cars (in thousands of dollars) advertised for sale at three different car dealers. If you want to go to the dealer whose prices seem least expensive, which dealer would you go to? Use statistics from the displays to justify your answer.



Performance Expectations *Students are expected to:*	Explanatory Comments and Examples
M1.5.B Describe how linear transformations affect the center and spread of univariate data.	Examples: • A company decides to give every one of its employees a $5,000 raise. What happens to the mean and standard deviation of the salaries as a result? • A company decides to double each of its employee's salaries. What happens to the mean and standard deviation of the salaries as a result?
M1.5.C Make valid inferences and draw conclusions based on data.	Determine whether arguments based on data confuse association with causation. Evaluate the reasonableness of and make judgments about statistical claims, reports, studies, and conclusions Example: • Mr. Shapiro found that the amount of time his students spent doing mathematics homework is positively correlated with test grades in his class. He concluded that doing homework makes students' test scores higher. Is this conclusion justified? Explain any flaws in Mr. Shapiro's reasoning.

Mathematics 1

***M1.6. Core Content:** Numbers, expressions, and operations* *(Numbers, Operations, Algebra)*

Students see the number system extended to the real numbers represented by the number line. They use variables and expressions to solve problems from purely mathematical as well as applied contexts. They build on their understanding of and ability to compute with arithmetic operations and properties and expand this understanding to include the symbolic language of algebra. Students demonstrate this ability to write and manipulate a wide variety of algebraic expressions throughout high school mathematics as they apply algebraic procedures to solve problems.

Performance Expectations

Students are expected to:

M1.6.A Know the relationship between real numbers and the number line, and compare and order real numbers with and without the number line.

Explanatory Comments and Examples

Although a formal definition of real numbers is beyond the scope of Mathematics 1, students learn that every point on the number line represents a real number, either rational or irrational, and that every real number has its unique point on the number line. They locate, compare, and order real numbers on the number line.

Real numbers include those written in scientific notation or expressed as fractions, decimals, exponentials, or roots.

Examples:

- Without using a calculator, order the following on the number line:

  $\sqrt{82}$, 3π, 8.9, 9, $\frac{37}{4}$, 9.3×10^0

- A star's color gives an indication of its temperature and age. The chart shows four types of stars and the lowest temperature of each type.

Type	Lowest Temperature (in °F)	Color
A	1.35×10^4	Blue-White
B	2.08×10^4	Blue
G	9.0×10^3	Yellow
P	4.5×10^4	Blue

  List the temperatures in order from lowest to highest.

Performance Expectations	*Explanatory Comments and Examples*

Students are expected to:

M1.6.B Determine whether approximations or exact values of real numbers are appropriate, depending on the context, and justify the selection.

Decimal approximations of numbers are sometimes used in applications such as carpentry or engineering, while at other times, these applications may require exact values. Students should understand the difference and know that the appropriate approximation depends upon the necessary degree of precision needed in given situations.

For example, 1.414 is an approximation and not an exact solution to the equation $x^2 - 2 = 0$, but $\sqrt{2}$ is an exact solution to this equation.

Example:

- Using a common engineering formula, an engineering student represented the maximum safe load of a bridge to be $1000(99 - 70\sqrt{2})$ tons.

  He used 1.41 as the approximation for $\sqrt{2}$ in his calculations. When the bridge was built and tested in a computer simulation to verify its maximum weight-bearing load, it collapsed! The student had estimated the bridge would hold ten times the weight that was applied to it when it collapsed.

  - — Calculate the weight that the student thought the bridge could bear using 1.41 as the estimate for $\sqrt{2}$.
  - — Calculate other weight values using estimates of $\sqrt{2}$ that have more decimal places. What might be a reasonable degree of precision required to know how much weight the bridge can handle safely? Justify your answer.

M1.6.C Recognize the multiple uses of variables, determine all possible values of variables that satisfy prescribed conditions, and evaluate algebraic expressions that involve variables.

Students learn to use letters as variables and in other ways that increase in sophistication throughout high school. For example, students learn that letters can be used:

- To represent fixed and temporarily unknown values in equations, such as $3x + 2 = 5$;
- To express identities, such as $x + x = 2x$ for all x;
- As attributes in formulas, such as $A = lw$;
- As constants such as a, b, and c in the equation $y = ax^2 + bx + c$;
- As parameters in equations, such as the m and b for the family of functions defined by $y = mx + b$;
- To represent varying quantities, such as x in $f(x) = 5x$;
- To represent functions, such as f in $f(x) = 5x$; and
- To represent specific numbers, such as π.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M1.6.C cont.	Expressions include those involving polynomials, radicals, absolute values, and integer exponents. Examples: • For what values of *a* and *n*, where *n* is an integer greater than 0, is a^n always negative? • For what values of *a* is $\frac{1}{a}$ an integer? • For what values of *a* is $\sqrt{5-a}$ defined? • For what values of *a* is -*a* always positive?
M1.6.D Solve an equation involving several variables by expressing one variable in terms of the others.	Examples: • Solve $A = p + prt$ for *p*. • Solve $V = \pi r^2 h$ for *h* or for *r*.

Mathematics 1

M1.7. Additional Key Content *(Numbers, Algebra)*

Students develop a basic understanding of arithmetic and geometric sequences and of exponential functions, including their graphs and other representations. They use exponential functions to analyze relationships, represent and model problems, and answer questions in situations that are modeled by these nonlinear functions. Students learn graphical and numerical methods for approximating solutions to exponential equations. Students interpret the meaning of problem solutions and explain limitations related to solutions.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M1.7.A Sketch the graph for an exponential function of the form $y = ab^n$ where n is an integer, describe the effects that changes in the parameters a and b have on the graph, and answer questions that arise in situations modeled by exponential functions.	Examples: • Sketch the graph of $y = 2^n$ by hand. • You have won a door prize and are given a choice between two options: $150 invested for 10 years at 4% compounded annually. $200 invested for 10 years at 3% compounded annually. How much is each worth at the end of each year of the investment periods? Are the two investments ever equal in value? Which will you choose?
M1.7.B Find and approximate solutions to exponential equations.	Students can approximate solutions using graphs or tables with and without technology.
M1.7.C Interpret and use integer exponents and square and cube roots, and apply the laws and properties of exponents to simplify and evaluate exponential expressions.	Examples: • $2^{-3} = \frac{1}{2^3}$ • $\frac{2^{-2}3^{2}5}{2^{2}3^{-3}5^{2}} = \frac{3^5}{2^4 5}$ • $\frac{a^{-2}b^{2}c}{a^{2}b^{-3}c^{2}} = \frac{b^5}{a^4 c}$ • $\sqrt{8} = \sqrt{2 \bullet 2 \bullet 2} = 2\sqrt{2}$ • $\sqrt[3]{a \bullet b} = \sqrt[3]{a} \bullet \sqrt[3]{b}$

Performance Expectations

Students are expected to:

M1.7.D Express arithmetic and geometric sequences in both explicit and recursive forms, translate between the two forms, explain how rate of change is represented in each form, and use the forms to find specific terms in the sequence.

Explanatory Comments and Examples

Examples:

- Write a recursive formula for the arithmetic sequence 5, 9, 13, 17, . . . What is the slope of the line that contains the points associated with these values and their position in the sequence? How is the slope of the line related to the sequence?
- Given that $u(0) = 3$ and $u(n + 1) = u(n) + 7$ when n is a positive integer,
  - a. find $u(5)$;
  - b. find n so that $u(n) = 361$; and
  - c. find a formula for $u(n)$.
- Write a recursive formula for the geometric sequence 5, 10, 20, 40, . . . and determine the 100^{th} term.
- Given that $u(0) = 2$ and $u(n + 1) = 3u(n)$,
  - a. find $u(4)$, and
  - b. find a formula for $u(n)$.

Mathematics 1

M1.8. Core Processes: *Reasoning, problem solving, and communication*

Students formalize the development of reasoning in Mathematics 1 as they use algebra, geometry, and statistics to make and defend generalizations. They justify their reasoning with accepted standards of mathematical evidence and proof, using correct mathematical language, terms, and symbols in all situations. They extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to mathematical and applied situations. Students formalize a coherent problem-solving process in which they analyze the situation to determine the question(s) to be answered, synthesize given information, and identify implicit and explicit assumptions that have been made. They examine their solution(s) to determine reasonableness, accuracy, and meaning in the context of the original problem. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an increasing amount of information is presented in quantitative ways and more and more occupations and fields of study rely on mathematics.

Performance Expectations

Students are expected to:

M1.8.A Analyze a problem situation and represent it mathematically.

M1.8.B Select and apply strategies to solve problems.

M1.8.C Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.

M1.8.D Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.

M1.8.E Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.

M1.8.F Summarize mathematical ideas with precision and efficiency for a given audience and purpose.

M1.8.G Synthesize information to draw conclusions, and evaluate the arguments and conclusions of others.

M1.8.H Use inductive reasoning to make conjectures, and use deductive reasoning to prove or disprove conjectures.

Explanatory Comments and Examples

Examples:

- Three teams of students independently conducted experiments to relate the rebound height of a ball to the rebound number. The table gives the average of the teams' results.

Rebound Number	Rebound Height (cm)
0	200
1	155
2	116
3	88
4	66
5	50
6	44

  Construct a scatterplot of the data, and describe the function that relates the height of the ball to the rebound number. Predict the rebound height of the ball on the tenth rebound. Justify your answer.

- Prove formally that the sum of two odd numbers is always even.

MATHEMATICS 2
STANDARDS

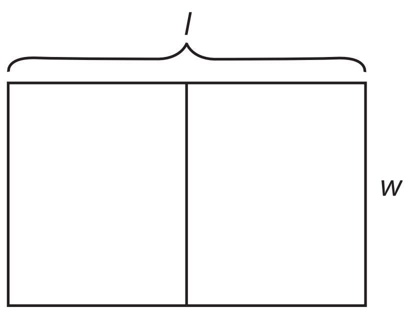
Mathematics 2

Mathematics 2 extends the study of functions to include quadratic functions, providing tools for modeling a greater variety of real-world situations. Students develop computational and algebraic skills that support analysis of these functions and their multiple representations. Students extend their ability to reason mathematically. They distinguish between inductive and deductive thinking, make conjectures, and prove theorems. Students become skilled in writing more involved proofs through their study of triangles, lines, and quadrilaterals. Finally, the study of probability extends students' understanding of proportional reasoning and relationships with the inclusion of counting methods and lays the groundwork for the study of data and variability in the next course.

Mathematics 2

M2.1. Core Content: *Modeling situations and solving problems* *(Algebra)*

This first core content area highlights the types of problems students will be able to solve by the end of Mathematics 2. Students extend their ability to model situations and solve problems with additional functions and equations in this course. Additionally, they deepen their understanding and proficiency with functions they encountered in Mathematics 1 and use these functions to solve more complex problems. When presented with a word problem, students determine which function or equation models the problem and then use that information to write an equation to solve the problem. Turning word problems into equations that can be solved is a skill students hone throughout the course.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M2.1.A Select and justify functions and equations to model and solve problems.	Example: • Dawson wants to make a horse corral by creating a rectangle that is divided into 2 parts, similar to the following diagram. He has a 1200-foot roll of fencing to do the job. What are the dimensions of the enclosure with the largest total area? What function or equation best models this situation? 
M2.1.B Solve problems that can be represented by systems of equations and inequalities.	Example: • Data derived from an experiment seems to be parabolic when plotted on a coordinate grid. Three observed data points are (2, 10), (3, 8), and (4, 4). Write a quadratic equation that passes through the points.
M2.1.C Solve problems that can be represented by quadratic functions, equations, and inequalities.	Students solve problems by factoring and applying the quadratic formula to the quadratic equation, and use the vertex form of the equation to solve problems about maximums, minimums, and symmetry. Examples: • Find the solutions to the simultaneous equations $y = x + 2$ and $y = x^2$.

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
M2.1.C cont.	• If you throw a ball straight up (with initial height of 4 feet) at 10 feet per second, how long will it take to fall back to the starting point? The function $h(t) = -16t^2 + v_0t + h_0$ describes the height, h in feet, of an object after t seconds, with initial velocity v_0 and initial height h_0. • Joe owns a small plot of land 20 feet by 30 feet. He wants to double the area by increasing both the length and the width, keeping the dimensions in the same proportion as the original. What will be the new length and width? • What two consecutive numbers, when multiplied together, give the first number plus 16? Write the equation that represents the situation. • The Gateway Arch in St. Louis has a special shape called a catenary, which looks a lot like a parabola. It has a base width of 600 feet and is 630 feet high. Which is taller, this catenary arch or a parabolic arch that has the same base width but has a height of 450 feet at a point 150 feet from one of the pillars? What is the height of the parabolic arch?
M2.1.D Solve problems that can be represented by exponential functions and equations.	Students extend their use of exponential functions and equations to solve more complex problems. They approximate solutions with graphs or tables, check solutions numerically, and when possible, solve problems exactly. Examples: • E. coli bacteria reproduce by a simple process called binary fission—each cell increases in size and divides into two cells. In the laboratory, E. coli bacteria divide approximately every 15 minutes. A new E. coli culture is started with 1 cell. a. Find a function that models the E. coli population size at the end of each 15-minute interval. Justify the function you found. b. State an appropriate domain for the model based on the context. c. After what 15-minute interval will you have at least 500 bacteria? • Estimate the solution to $2^x = 16,384$

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
M2.1.E Solve problems involving combinations and permutations.	Examples: • The company Ali works for allows her to invest in her choice of 10 different mutual funds, 6 of which grew by at least 5% over the last year. Ali randomly selected 4 of the 10 funds in which to invest. What is the probability that 3 of Ali's funds grew by 5%? • Four points (*A*, *B*, *C*, and *D*) lie on one straight line, *n*, and five points (*E*, *F*, *G*, *H*, and *J*) lie on another straight line, *m*, that is parallel to *n*. What is the probability that three points, selected at random, will form a triangle?

Mathematics 2

M2.2. Core Content: *Quadratic functions, equations, and relationships* *(Algebra)*

Students learn that exponential and quadratic functions can be used to model some situations where linear functions may not be the best model. They use graphical and numerical methods with exponential functions of the form $y = ab^x$ and quadratic functions to analyze relationships, represent and model problems, and answer questions. Students extend their algebraic skills and learn various methods of solving quadratic equations over real or complex numbers, including using the quadratic formula, factoring, graphing, and completing the square. They learn to translate between forms of quadratic equations, applying the vertex form to evaluate maximum and minimum values and find symmetry of the graph, and they learn to identify which form should be used in a particular situation. They interpret the meaning of problem solutions and explain their limitations. Students recognize common examples of situations that can be modeled by quadratic functions, such as maximizing area or the height of an object moving under the force of gravity. They compare the characteristics of quadratic functions to those of linear and exponential functions. The understanding of these particular types of functions, together with students' understanding of linear functions, provides students with a powerful set of tools to use mathematical models to deal with problems and situations in advanced mathematics courses, in the workplace, and in everyday life.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M2.2.A Represent a quadratic function with a symbolic expression, as a graph, in a table, and with a description, and make connections among the representations.	Example: • Kendre and Tyra built a tennis ball cannon that launches tennis balls straight up in the air at an initial velocity of 50 feet per second. The mouth of the cannon is 2 feet off the ground. The function $h(t) = -16t^2 + 50t + 2$ describes the height, h, in feet, of the ball t seconds after the launch. Make a table from the function. Then use the table to sketch a graph of the height of the tennis ball as a function of time into the launch. Give a verbal description of the graph. How high was the ball after 1 second? When does it reach this height again?
M2.2.B Sketch the graph of a quadratic function, describe the effects that changes in the parameters have on the graph, and interpret the x-intercepts as solutions to a quadratic equation.	Note that in Mathematics 2, the parameter b in the term bx in the quadratic form $ax^2 + bx + c$ is not often used to provide useful information about the characteristics of the graph. Parameters considered most useful are: • a and c in $f(x) = ax^2 + c$ • a, h, and k in $f(x) = a(x – h)^2 + k$, and • a, r, and s in $f(x) = a(x – r)(x – s)$

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
M2.2.B cont.	Example: • A particular quadratic function can be expressed in the following two ways: $f(x) = -(x - 3)^2 + 1$ $f(x) = -(x - 2)(x - 4)$ What information about the graph can be directly inferred from each of these forms? Explain your reasoning. Sketch the graph of this function, showing the roots.
M2.2.C Translate between the standard form of a quadratic function, the vertex form, and the factored form; graph and interpret the meaning of each form.	Students translate among forms of a quadratic function to convert to one that is appropriate—e.g., vertex form—to solve specific problems. Students learn about the advantages of the standard form ($f(x) = ax^2 + bx + c$), the vertex form ($f(x) = a(x - h)^2 + d$), and the factored form ($f(x) = a(x - r)(x - s)$). They produce the vertex form by completing the square on the function in standard form, which allows them to see the symmetry of the graph of a quadratic function as well as the maximum or minimum. This opens up a whole range of new problems students can solve using quadratics. Students continue to find the solutions of the equation, which can be either real or complex. Example: • Find the minimum, the line of symmetry, and the roots for the graphs of each of the following functions: $f(x) = x^2 - 4x + 3$ $f(x) = x^2 - 4x + 4$ $f(x) = x^2 - 4x + 5$
M2.2.D Solve quadratic equations that can be factored as $(ax + b)(cx + d)$ where a, b, c, and d are integers.	Students learn to efficiently solve quadratic equations by recognizing and using the simplest factoring methods, including recognizing special quadratics as squares and differences of squares. Examples: • $2x^2 + x - 3 = 0$; $(x - 1)(2x + 3) = 0$; $x = 1, -\frac{3}{2}$ • $4x^2 + 6x = 0$; $2x(2x + 3) = 0$; $x = 0, -\frac{3}{2}$ • $36x^2 - 25 = 0$; $(6x + 5)(6x - 5) = 0$; $x = \pm\frac{5}{6}$ • $x^2 + 6x + 9 = 0$; $(x + 3)^2 = 0$; $x = -3$

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
M2.2.E Determine the number and nature of the roots of a quadratic function.	Students should be able to recognize and interpret the discriminant. Students should also be familiar with the Fundamental Theorem of Algebra, i.e., that all polynomials, not just quadratics, have roots over the complex numbers. This concept becomes increasingly important as students progress through mathematics. Example: • For what values of a does $f(x) = x^2 - 6x + a$ have 2 real roots, 1 real root, and no real roots?
M2.2.F Solve quadratic equations that have real roots by completing the square and by using the quadratic formula.	Students solve those equations that are not easily factored by completing the square and by using the quadratic formula. Completing the square should also be used to derive the quadratic formula. Students learn how to determine if there are two, one, or no real solutions. Examples: • Complete the square to solve $x^2 + 4x = 13$. $x^2 + 4x - 13 = 0$ $x^2 + 4x + 4 = 17$ $(x + 2)^2 = 17$ $x + 2 = \pm\sqrt{17}$ $x = -2 \pm\sqrt{17}$ $x \approx 2.12, -6.12$ • Use the quadratic formula to solve $4x^2 - 2x = 5$. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(4 \bullet -5)}}{2(4)}$ $x = \dfrac{2 \pm \sqrt{84}}{8}$ $x = \dfrac{2 \pm 2\sqrt{21}}{8}$ $x = \dfrac{1 \pm \sqrt{21}}{4}$ $x \approx 1.40, -0.90$

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M2.2.G Solve quadratic equations and inequalities, including equations with complex roots.	Students solve equations that are not easily factored by completing the square and by using the quadratic formula. Examples: • $x^2 - 10x + 34 = 0$ • $3x^2 + 10 = 4x$ • Wile E. Coyote launches an anvil from 180 feet above the ground at time $t = 0$. The equation that models this situation is given by $h = -16t^2 + 96t + 180$, where t is time measured in seconds and h is height above the ground measured in feet. a. What is a reasonable domain restriction for t in this context? b. Determine the height of the anvil two seconds after it was launched. c. Determine the maximum height obtained by the anvil. d. Determine the time when the anvil is more than 100 feet above ground. • Farmer Helen wants to build a pigpen. With 100 feet of fence, she wants a rectangular pen with one side being a side of her existing barn. What dimensions should she use for her pigpen in order to have the maximum number of square feet?
M2.2.H Determine if a bivariate data set can be better modeled with an exponential or a quadratic function and use the model to make predictions.	In high school, determining a formula for a curve of best fit requires a graphing calculator or similar technological tool.

Mathematics 2

***M2.3. Core Content:** Conjectures and proofs* *(Algebra, Geometry/Measurement)*

Students extend their knowledge of two-dimensional geometric figures and their properties to include quadrilaterals and other polygons, with special emphasis on necessary and sufficient conditions for triangle congruence. They work with geometric constructions, using dynamic software as a tool for exploring geometric relationships and formulating conjectures and using compass-and-straightedge and paper-folding constructions as contexts in which students demonstrate their knowledge of geometric relationships. Students define the basic trigonometric ratios and use them to solve problems in a variety of applied situations. They formalize the reasoning skills they have developed in previous grades and solidify their understanding of what it means to mathematically prove a geometric statement. Students encounter the concept of formal proof built on definitions, axioms, and theorems. They use inductive reasoning to test conjectures about geometric relationships and use deductive reasoning to prove or disprove their conclusions. Students defend their reasoning using precise mathematical language and symbols. Finally, they apply their knowledge of linear functions to make and prove conjectures about geometric figures on the coordinate plane.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M2.3.A Use deductive reasoning to prove that a valid geometric statement is true.	Valid proofs may be presented in paragraph, two-column, or flow-chart formats. Proof by contradiction is a form of deductive reasoning. Example: • Prove that the diagonals of a rhombus are perpendicular bisectors of each other.
M2.3.B Identify errors or gaps in a mathematical argument and develop counterexamples to refute invalid statements about geometric relationships.	Example: • Identify errors in reasoning in the following proof: Given $\angle ABC \cong \angle PRQ$, $\overline{AB} \cong \overline{PQ}$, and $\overline{BC} \cong \overline{QR}$, then $\Delta ABC \cong \Delta PQR$ by SAS.
M2.3.C Write the converse, inverse, and contrapositive of a valid proposition and determine their validity.	Examples: • If m and n are odd integers, then the sum of m and n is an even integer. State the converse and determine whether it is valid. • If a quadrilateral is a rectangle, the diagonals have the same length. State the contrapositive and determine whether it is valid.
M2.3.D Distinguish between definitions and undefined geometric terms and explain the role of definitions, undefined terms, postulates (axioms), and theorems.	Students sketch *points* and *lines* (undefined terms) and define and sketch representations of other common terms. They use definitions and postulates as they prove theorems throughout geometry. In their work with theorems, they identify the hypothesis and the conclusion and explain the role of each. Students describe the consequences of changing assumptions or using different definitions for subsequent theorems and logical arguments.

Performance Expectations

Students are expected to:

Explanatory Comments and Examples

M2.3.D cont.

Example:

- There are two definitions of trapezoid that can be found in books or on the web. A trapezoid is either
  - — a quadrilateral with exactly one pair of parallel sides or
  - — a quadrilateral with at least one pair of parallel sides.

  Write some theorems that are true when applied to one definition but not the other, and explain your answer.

M2.3.E Know, explain, and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle.

Examples:

- Prove that the sum of the angles of a triangle is 180°.
- Prove and explain theorems about the incenter, circumcenter, orthocenter, and centroid.
- The rural towns of Atwood, Bridgeville, and Carnegie are building a communications tower to serve the needs of all three towns. They want to position the tower so that the distance from each town to the tower is equal. Where should they locate the tower? How far will it be from each town?

M2.3.F Determine and prove triangle congruence and other properties of triangles.

Students extend their work with similarity in Mathematics 1 to proving theorems about congruence and other properties of triangles.

Students should identify necessary and sufficient conditions for congruence in triangles, and use these conditions in proofs.

Examples:

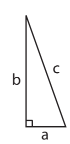
- Prove that congruent triangles are similar.
- Show that not all SSA triangles are congruent.

M2.3.G Know, prove, and apply the Pythagorean Theorem and its converse.

Students extend their work with the Pythagorean Theorem from previous grades to include formal proof.

Examples:

- Consider any right triangle with legs *a* and *b* and hypotenuse *c*. The right triangle is used to create Figures 1 and 2. Explain how these figures constitute a visual representation of a proof of the Pythagorean Theorem.



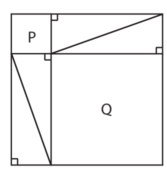


Figure 1

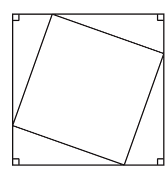
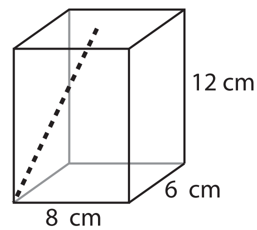
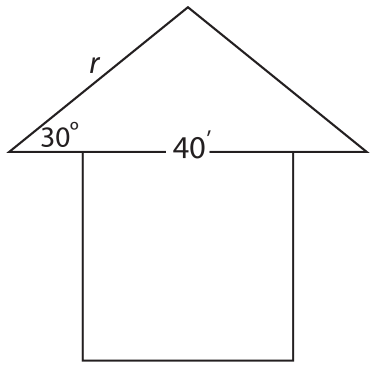


Figure 2

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
M2.3.G cont.	• A juice box is 6 cm by 8 cm by 12 cm. A straw is inserted into a hole in the center of the top of the box. The straw must stick out 2 cm so you can drink from it. If the straw must be long enough to touch each bottom corner of the box, what is the minimum length the straw must be? (Assume the diameter of the straw is 0 for the mathematical model.)  • In ΔABC, with right angle at *C*, draw the altitude $\overline{CD}$ from C to $\overline{AB}$. Name all similar triangles in the diagram. Use these similar triangles to prove the Pythagorean Theorem. • Apply the Pythagorean Theorem to derive the distance formula in the (*x*, *y*) plane. • Determine the length of the altitude of an equilateral triangle whose side lengths measure 5 units.
M2.3.H Solve problems involving the basic trigonometric ratios of sine, cosine, and tangent.	Students apply their knowledge of the Pythagorean Theorem from Grade 8 to define the basic trigonometric ratios. They formally prove the Pythagorean Theorem in Mathematics 2. Examples: • A 12-foot ladder leans against a wall to form a 63° angle with the ground. How many feet above the ground is the point on the wall at which the ladder is resting? • Use the Pythagorean Theorem to establish that $\sin^2 ø + \cos^2 ø = 1$ for ø between 0° and 90°.

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
M2.3.I Use the properties of special right triangles (30°–60°–90° and 45°–45°–90°) to solve problems.	Examples: • If one leg of a right triangle has length 5 and the adjacent angle is 30°, what is the length of the other leg and the hypotenuse? • If one leg of a 45°–45°–90° triangle has length 5, what is the length of the hypotenuse? • The pitch of a symmetrical roof on a house 40 feet wide is 30º. What is the length of the rafter, *r*, exactly and approximately? 
M2.3.J Know, prove, and apply basic theorems about parallelograms.	Properties may include those that address symmetry and properties of angles, diagonals, and angle sums. Students may use inductive and deductive reasoning and counterexamples. Examples: • Are opposite sides of a parallelogram always congruent? Why or why not? • Are opposite angles of a parallelogram always congruent? Why or why not? • Prove that the diagonals of a parallelogram bisect each other. • Explain why if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. • Prove that the diagonals of a rectangle are congruent. Is this true for any parallelogram? Justify your reasoning.

Performance Expectations

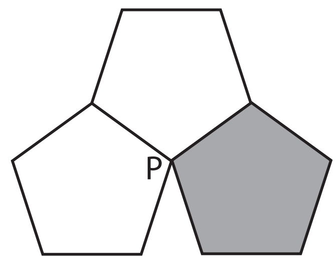
Students are expected to:

M2.3.K Know, prove, and apply theorems about properties of quadrilaterals and other polygons.

Explanatory Comments and Examples

Examples:

- What is the length of the apothem of a regular hexagon with side length 8? What is the area of the hexagon?
- If the shaded pentagon were removed, it could be replaced by a regular *n*-sided polygon that would exactly fill the remaining space. Find the number of sides, *n*, of a replacement polygon that makes the three polygons fit perfectly.



M2.3.L Determine the coordinates of a point that is described geometrically.

Examples:

- Determine the coordinates for the midpoint of a given line segment
- Given the coordinates of three vertices of a parallelogram, determine all possible coordinates for the fourth vertex.
- Given the coordinates for the vertices of a triangle, find the coordinates for the center of the circumscribed circle and the length of its radius.

Performance Expectations

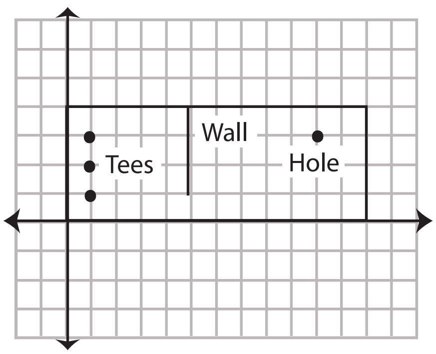
Students are expected to:

M2.3.M Verify and apply properties of triangles and quadrilaterals in the coordinate plane.

Explanatory Comments and Examples

Examples:

- Given four points in a coordinate plane that are the vertices of a quadrilateral, determine whether the quadrilateral is a rhombus, a square, a rectangle, a parallelogram, or none of these. Name all the classifications that apply.
- Given a parallelogram on a coordinate plane, verify that the diagonals bisect each other.
- Given the line with *y*-intercept 4 and *x*-intercept 3, find the area of a square that has one corner on the origin and the opposite corner on the line described.
- Below is a diagram of a miniature golf hole as drawn on a coordinate grid. The dimensions of the golf hole are 4 feet by 12 feet. Players must start their ball from one of the three tee positions, located at (1, 1), (1, 2), or (1, 3). The hole is located at (10, 3). A wall separates the tees from the hole. At which tee should the ball be placed to create the shortest "hole-in-one" path? Sketch the intended path of the ball, find the distance the ball will travel, and describe your reasoning. (Assume the diameters of the golf ball and the hole are 0 for the mathematical model.)



Mathematics 2

M2.4. Core Content: *Probability* *(Data/Statistics/Probability)*

Students formalize their study of probability, computing both combinations and permutations to calculate the likelihood of an outcome in uncertain circumstances and applying the binominal theorem to solve problems. They apply their understanding of probability to a wide range of practical situations, including those involving permutations and combinations. Understanding probability helps students become knowledgeable consumers who make sound decisions about high-risk games, financial issues, etc.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M2.4.A Apply the fundamental counting principle and the ideas of order and replacement to calculate probabilities in situations arising from two-stage experiments (compound events).	
M2.4.B Given a finite sample space consisting of equally likely outcomes and containing events A and B, determine whether A and B are independent or dependent, and find the conditional probability of A given B.	Example: • What is the probability of drawing a heart from a standard deck of cards on a second draw, given that a heart was drawn on the first draw and not replaced?
M2.4.C Compute permutations and combinations, and use the results to calculate probabilities.	Example: • Two friends, Abby and Ben, are among five students being considered for three student council positions. If each of the five students has an equal likelihood of being selected, what is the probability that Abby and Ben will both be selected?
M2.4.D Apply the binomial theorem to solve problems involving probability.	The binominal theorem is also applied when computing with polynomials. Examples: • Use Pascal's triangle and the binomial theorem to find the number of ways six objects can be selected four at a time. • In a survey, 33% of adults reported that they preferred to get the news from newspapers rather than television. If you survey 5 people, what is the probability of getting exactly 2 people who say they prefer news from the newspaper? Write an equation that can be used to solve the problem. Create a histogram of the binomial distribution of the probability of getting 0 through 5 responders saying they prefer the newspaper.

Mathematics 2

M2.5. Additional Key Content *(Algebra, Measurement)*

Students grow more proficient in their use of algebraic techniques as they use these techniques to write equivalent expressions in various forms. They build on their understanding of computation using arithmetic operations and properties and expand this understanding to include the symbolic language of algebra. Students understand the role of units in measurement, convert among units within and between different measurement systems as needed, and apply what they know to solve problems. They use derived measures such as those used for speed (e.g., feet per second) or determining automobile gas consumption (e.g., miles per gallon).

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M2.5.A Use algebraic properties to factor and combine like terms in polynomials.	Algebraic properties include the commutative, associative, and distributive properties. Factoring includes: • Factoring a monomial from a polynomial, such as $4x^2 + 6x = 2x(2x + 3)$; • Factoring the difference of two squares, such as $36x^2 – 25y^2 = (6x + 5y)(6x – 5y)$ and $x^4 – y^4 = (x + y)(x – y)(x^2 + y^2)$; • Factoring perfect square trinomials, such as $x^2 + 6xy + 9y^2 = (x + 3y)^2$; • Factoring quadratic trinomials such as $x^2 + 5x + 4 = (x + 4)(x + 1)$; and • Factoring trinomials that can be expressed as the product of a constant and a trinomial, such as $0.5x^2 – 2.5x – 7 = 0.5(x + 2)(x – 7)$.
M2.5.B Use different degrees of precision in measurement, explain the reason for using a certain degree of precision, and apply estimation strategies to obtain reasonable measurements with appropriate precision for a given purpose.	Example: • The U.S. Census Bureau reported a national population of 299,894,924 on its Population Clock in mid-October of 2006. One can say that the U.S. population is 3 hundred million (3×10^8) and be precise to one digit. Although the population had surpassed 3 hundred million by the end of that month, explain why 3×10^8 remained precise to one digit.
M2.5.C Solve problems involving measurement conversions within and between systems, including those involving derived units, and analyze solutions in terms of reasonableness of solutions and appropriate units.	This performance expectation is intended to build on students' knowledge of proportional relationships. Students should understand the relationship between scale factors and their inverses as they relate to choices about when to multiply and when to divide in converting measurements. Derived units include those that measure speed, density, flow rates, population density, etc.

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
M2.5.C cont.	Example: • A digital camera takes pictures that are 3.2 megabytes in size. If the pictures are stored on a 1-gigabyte card, how many pictures can be taken before the card is full?
M2.5.D Find the terms and partial sums of arithmetic and geometric series and the infinite sum for geometric series.	Students build on the knowledge gained in Mathematics 1 to find specific terms in a sequence and to express arithmetic and geometric sequences in both explicit and recursive forms. Examples: • A ball is dropped from a height of 10 meters. Each time it hits the ground, it rebounds $\frac{3}{4}$ of the distance it has fallen. What is the total sum of the distances it falls and rebounds before coming to rest? • Show that the sum of the first 10 terms of the geometric series $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$ is twice the sum of the first 10 terms of the geometric series $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots$

Mathematics 2

M2.6. Core Processes: *Reasoning, problem solving, and communication*

Students formalize the development of reasoning in Mathematics 2 as they use algebra, geometry, and probability to make and defend generalizations. They justify their reasoning with accepted standards of mathematical evidence and proof, using correct mathematical language, terms, and symbols in all situations. They extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to mathematical and applied situations. Students formalize a coherent problem-solving process in which they analyze the situation to determine the question(s) to be answered, synthesize given information, and identify implicit and explicit assumptions that have been made. They examine their solution(s) to determine reasonableness, accuracy, and meaning in the context of the original problem. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an increasing amount of information is presented in quantitative ways and more and more occupations and fields of study rely on mathematics.

Performance Expectations

Students are expected to:

M2.6.A Analyze a problem situation and represent it mathematically.

M2.6.B Select and apply strategies to solve problems.

M2.6.C Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.

M2.6.D Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.

M2.6.E Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.

M2.6.F Summarize mathematical ideas with precision and efficiency for a given audience and purpose.

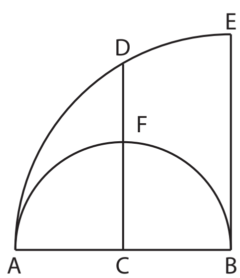
M2.6.G Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.

M2.6.H Use inductive reasoning to make conjectures, and use deductive reasoning to prove or disprove conjectures.

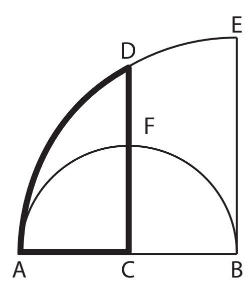
Explanatory Comments and Examples

Examples:

- $\overline{AB}$ is the diameter of the semicircle and the radius of the quarter circle shown in the figure below. $\overline{BC}$ is the perpendicular bisector of $\overline{AB}$.



Imagine all of the triangles formed by $\overline{AB}$ and any arbitrary point lying in the region bounded by $\overline{AC}$, $\overline{CD}$, and $\overset{\frown}{AD}$, seen in bold below.



Use inductive reasoning to make conjectures about what types of triangles are formed based upon the region where the third vertex is located. Use deductive reasoning to verify your conjectures.

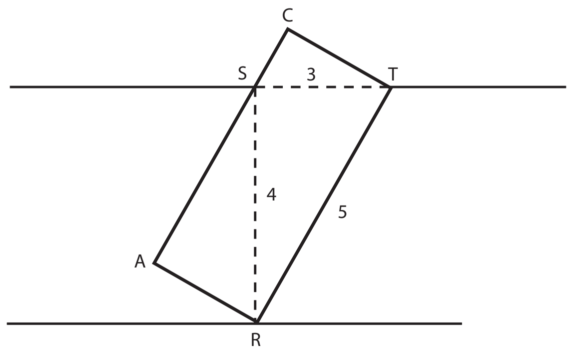
Performance Expectations

Students are expected to:

M2.6 cont.

Explanatory Comments and Examples

- Rectangular cartons that are 5 feet long need to be placed in a storeroom that is located at the end of a hallway. The walls of the hallway are parallel. The door into the hallway is 3 feet wide and the width of the hallway is 4 feet. The cartons must be carried face up. They may not be tilted. Investigate the width and carton top area that will fit through the doorway.



Generalize your results for a hallway opening of x feet and a hallway width of y feet if the maximum carton dimensions are c feet long and $x^2 + y^2 = c^2$.

- Prove $(a + b)^2 = a^2 + 2ab + b^2$.
- A student writes $(x + 3)^2 = x^2 + 9$. Explain why this is incorrect.

MATHEMATICS 3
STANDARDS

Mathematics 3

In Mathematics 3, students develop a more coherent and formal view of mathematics, going beyond specific rules and procedures to emphasize generalizations. Students extend their knowledge of number systems to include complex numbers, and they evaluate possible solutions to algebraic equations. The application and visualization of geometry extends to three-dimensional figures as students study the effects of changes in one dimension on various attributes and properties of a figure. Students study the composition of transformations on geometric figures. They generalize the relationship of changes in the symbolic form of functions to transformations of their corresponding graphs. They extend their study of functions to include logarithmic, radical, and cubic functions and are introduced to the concept of inverse functions. Students study variability of data and examine the validity of generalizing results to an entire population.

Mathematics 3

***M3.1. Core Content:** Solving problems* *(Algebra)*

The first core content area highlights the types of problems students will be able to solve by the end of Mathematics 3, as they extend their ability to solve problems with additional functions and equations. Additionally, they deepen their understanding of and skills related to functions they encountered in Mathematics 1 and 2, and they use these functions to solve more complex problems. When presented with a contextual problem, students identify a function or equation that models the problem and use that information to write an equation to solve the problem. For example, in addition to using graphs to approximate solutions to problems modeled by exponential functions, they use knowledge of logarithms to solve exponential equations. Turning word problems into equations that can be solved is a skill students hone throughout the course.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M3.1.A Select and justify functions and equations to model and solve problems.	Examples: • A manufacturer wants to design a cylindrical soda can that will hold 500 milliliters (mL) of soda. The manufacturer's research has determined that an optimal can height is between 10 and 15 centimeters. Find a function for the radius in terms of the height, and use it to find the possible range of radius measurements in centimeters. Explain your reasoning.
M3.1.B Solve problems that can be represented by systems of equations and inequalities.	Examples: • Mr. Smith uses the following formula to calculate students' final grades in his Mathematics 3 class: $0.4E + 0.6T = C$, where E represents the score on the final exam, and T represents the average score of all tests given during the grading period. All tests and the final exam are worth a maximum of 100 points. The minimum passing score on tests, the final exam, and the course is 60. Determine the inequalities that describe the following situation and sketch a system of graphs to illustrate it. When necessary, round scores to the nearest tenth. — Is it possible for a student to have a failing test score average (i.e., $T < 60$ points) and still pass the course? — If you answered "yes," what is the minimum test score average a student can have and still pass the course? What final exam score is needed to pass the course with a minimum test score average? — A student has a particular test score average. How can (s)he figure out the minimum final exam score needed to pass the course?

Performance Expectations

Students are expected to:

M3.1.C Solve problems that can be represented by quadratic functions, equations, and inequalities.

Explanatory Comments and Examples

In addition to solving area and velocity problems by factoring and applying the quadratic formula to the quadratic equation, students use the vertex form of the equation to solve problems about maximums, minimums, and symmetry.

Examples:

- Fireworks are launched upward from the ground with an initial velocity of 160 feet per second. The formula for vertical motion is $h(t) = 0.5at^2 + vt + s$, where the gravitational constant, a, is -32 feet per square second, v represents the initial velocity, and s represents the initial height. Time t is measured in seconds, and height h is measured in feet.

  For the ultimate effect, the fireworks must explode after they reach the maximum height. For the safety of the crowd, they must explode at least 256 ft. above the ground. The fuses must be set for the appropriate time interval that allows the fireworks to reach this height. What range of times, starting from initial launch and ending with fireworks explosion, meets these conditions?

M3.1.D Solve problems that can be represented by exponential and logarithmic functions and equations.

Examples:

- If you need $15,000 in 4 years to start college, how much money would you need to invest now? Assume an annual interest rate of 4% compounded monthly for 48 months.
- The half-life of a certain radioactive substance is 65 days. If there are 4.7 grams initially present, how long will it take for there to be less than 1 gram of the substance remaining?

M3.1.E Solve problems that can be represented by inverse variations of the forms $f(x) = \frac{a}{x} + b$, $f(x) = \frac{a}{x^2} + \text{b}$, and $f(x) = \frac{a}{(bx+c)}$.

Examples:

- At the You're Toast, Dude! toaster company, the weekly cost to run the factory is $1400, and the cost of producing each toaster is an additional $4 per toaster.
  - — Find a function to represent the weekly cost in dollars, $C(x)$, of producing x toasters. Assume either unlimited production is possible or set a maximum per week.
  - — Find a function to represent the total production cost per toaster for a week.
  - — How many toasters must be produced within a week to have a total production cost per toaster of $8?

Performance Expectations

Students are expected to:

M3.1.E cont.

Explanatory Comments and Examples

- A person's weight varies inversely as the square of his distance from the center of the earth. Assume the radius of the earth is 4000 miles. How much would a 200-pound man weigh
  - — 1000 miles above the surface of the earth?
  - — 2000 miles above the surface of the earth?

Mathematics 3

***M3.2. Core Content:** Transformations and functions* *(Algebra, Geometry/Measurement)*

Students formalize their previous study of geometric transformations, focusing on the effect of such transformations on the attributes of geometric figures. They study techniques for using transformations to determine congruence and similarity. Students extend their study of transformations to include transformations of many types of functions, including quadratic and exponential functions.

Performance Expectations *Students are expected to:*		***Explanatory Comments and Examples***
M3.2.A	Sketch results of transformations and compositions of transformations for a given two-dimensional figure on the coordinate plane, and describe the rule(s) for performing translations or for performing reflections about the coordinate axes or the line $y = x$.	Transformations include translations, rotations, reflections, and dilations. Example: • Line *m* is described by the equation $y = 2x + 3$. Graph line *m* and reflect line *m* across the line $y = x$. Determine the equation of the image of the reflection. Describe the relationship between the line and its image.
M3.2.B	Determine and apply properties of transformations.	Students make and test conjectures about compositions of transformations and inverses of transformations, the commutativity and associativity of transformations, and the congruence and similarity of two-dimensional figures under various transformations. Examples: • Identify transformations (alone or in composition) that preserve congruence. • Determine whether the composition of two reflections of a line is commutative. • Determine whether the composition of two rotations about the same point of rotation is commutative. • Find a rotation that is equivalent to the composition of two reflections over intersecting lines. • Find the inverse of a given transformation.
M3.2.C	Given two congruent or similar figures in a coordinate plane, describe a composition of translations, reflections, rotations, and dilations that superimposes one figure on the other.	Examples: • Find a sequence of transformations that superimposes the segment with endpoints (0, 0) and (2, 1) on the segment with endpoints (4, 2) and (3, 0). • Find a sequence of transformations that superimposes the triangle with vertices (0, 0), (1, 1), and (2, 0) on the triangle with vertices (0, 1), (2, -1), and (0, -3).

Performance Expectations *Students are expected to:*	Explanatory Comments and Examples

M3.2.D Describe the symmetries of two-dimensional figures and describe transformations, including reflections across a line and rotations about a point.

Although the expectation only addresses two-dimensional figures, classroom activities can easily extend to three-dimensional figures. Students can also describe the symmetries, reflections across a plane, and rotations about a line for three-dimensional figures.

M3.2.E Construct new functions using the transformations $f(x – h)$, $f(x) + k$, $cf(x)$, and by adding and subtracting functions, and describe the effect on the original graph(s).

Students perform simple transformations on functions, including those that contain the absolute value of expressions, quadratic expressions, square root expressions, and exponential expressions, to make new functions.

Examples:

- What sequence of transformations changes $f(x) = x^2$ to $g(x) = -5(x – 3)^2 + 2$?
- Carly decides to earn extra money by making glass bead bracelets. She purchases tools for \$40.00. Elastic bead cord for each bracelet costs \$0.10. Glass beads come in packs of 10 beads, and one pack has enough beads to make one bracelet. Base price for the beads is \$2.00 per pack. For each of the first 100 packs she buys, she gets \$0.01 off each of the packs. (For example, if she purchases three packs, each pack costs \$1.97 instead of \$2.00.) Carly plans to sell each bracelet for \$4.00. Assume Carly will make a maximum of 100 bracelets.
    - — Find a function $C(b)$ that describes Carly's costs.
    - — Find a function $R(b)$ that describes Carly's revenue.

  Carly's profit is described by $P(b) = R(b) – C(b)$.
    - — Find $P(b)$.
    - — What is the minimum number of bracelets that Carly must sell in order to make a profit?
    - — To make a profit of \$100?

Mathematics 3

M3.3. Core Content: *Functions and modeling* *(Algebra)*

Students extend their understanding of exponential functions from Mathematics 2 with an emphasis on inverse functions. This leads to a natural introduction of logarithms and logarithmic functions. They learn to use the basic properties of exponential and logarithmic functions, graphing both types of functions to analyze relationships, represent and model problems, and answer questions. Students apply these functions in many practical situations, such as applying exponential functions to determine compound interest and applying logarithmic functions to determine the pH of a liquid. In addition, students extend their study of functions to include polynomials of higher degree and those containing radical expressions. They formalize and deepen their understanding of real-valued functions, their defining characteristics and uses, and the mathematical language used to describe them. They compare and contrast the types of functions they have studied and their basic transformations. Students learn the practical and mathematical limitations that must be considered when working with functions or when using functions to model situations.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M3.3.A Know and use basic properties of exponential and logarithmic functions and the inverse relationship between them.	Examples: • Given $f(x) = 4^x$, write an equation for the inverse of this function. Graph the functions on the same coordinate grid. — Find $f(-3)$. — Evaluate the inverse function at 7. • Derive the formulas: — $\log_b a \cdot \log_a b = 1$ — $\log_a N = \log_b N \cdot \log_a b$ • Find the exact value of x in: — $\log_x 16 = \frac{4}{3}$ — $\log_3 81 = x$ • Solve for y in terms of x: — $\log_a \frac{y}{x} = x$ — $100 = x \cdot 10^y$
M3.3.B Graph an exponential function of the form $f(x) = ab^x$ and its inverse logarithmic function.	Students expand on the work they did in Mathematics 2 with functions of the form $y = ab^x$. Although the concept of inverses is not fully developed until Precalculus, there is an emphasis in Mathematics 3 on students recognizing the inverse relationship between exponential and logarithmic functions and how this is reflected in the shapes of the graphs. Example: • Find the equation for the inverse function of $y = 3^x$. Graph both functions. What characteristics of each of the graphs indicate they are inverse functions?

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	

M3.3.C Solve exponential and logarithmic equations.

Examples:

- A recommended adult dosage of the cold medication NoMoreFlu is 16 mL. NoMoreFlu causes drowsiness when there are more than 4 mL in one's system, making it unsafe to drive, operate machinery, etc. The manufacturer wants to print a warning label telling people how long they should wait after taking NoMoreFlu for the drowsiness to pass. If the typical metabolic rate is such that one quarter of the NoMoreFlu is lost every four hours, and a person takes the full dosage, how long should adults wait after taking NoMoreFlu to ensure that there will be
  - — Less than 4 mL of NoMoreFlu in their system?
  - — Less than 1 mL in their system?
  - — Less than 0.1 mL in their system?
- Solve for x in $256 = 2^{x^2-1}$.
- Solve for x in $\log_5(x - 4) = 3$.

M3.3.D Plot points, sketch, and describe the graphs of functions of the form $f(x) = a\sqrt{x-c} + d$, and solve related equations.

Students solve algebraic equations that involve the square root of a linear expression over the real numbers. Students should be able to identify extraneous solutions and explain how they arose.

Students should view the function $g(x) = \sqrt{x}$ as the inverse function of $f(x) = x^2$, recognizing that the functions have different domains for x greater than or equal to 0.

Example:

- Analyze the following equations and tell what you know about the solutions. Then solve the equations.
  - — $2\sqrt{x+5} = 7$
  - — $\sqrt{5x-6} = -2$
  - — $\sqrt{2x+15} = x$
  - — $\sqrt{2x-5} = x+7$

M3.3.E Plot points, sketch, and describe the graphs of functions of the form $f(x) = \frac{a}{x^2} + b$, and $f(x) = \frac{a}{(bx+c)}$, and solve related equations.

Examples:

- Sketch the graphs of the four functions $f(x) = \frac{a}{x^2} + b$ when a = 4 and 8 and b = 0 and 1.
- Sketch the graphs of the four functions $f(x) = \frac{4}{(bx+c)}$ when b = 1 and 4 and c = 2 and 3.

Performance Expectations

Students are expected to:

M3.3.F Plot points, sketch, and describe the graphs of cubic polynomial functions of the form $f(x) = ax^3 + d$ as an example of higher order polynomials and solve related equations.

M3.3.G Solve systems of three equations with three variables.

Explanatory Comments and Examples

Example:

- Solve for x in $60 = -2x^3 + 6$.

Students solve systems of equations using algebraic and numeric methods.

Examples:

- Jill, Ann, and Stan are to inherit $20,000. Stan is to get twice as much as Jill, and Ann is to get twice as much as Stan. How much does each get?
- Solve the following system of equations.

  $2x – y – z = 7$

  $3x + 5y + z = -10$

  $4x – 3y + 2z = 4$

Mathematics 3

***M3.4. Core Content:** Quantifying variability* *(Data/Statistics/Probability)*

Students extend their use of statistics as they graph bivariate data and analyze its shape to make predictions. They calculate and interpret measures of variability, confidence intervals, and margins of error for population proportions. Dual goals underlie the content in the section: Students prepare for the further study of statistics and also become thoughtful consumers of data.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M3.4.A Calculate and interpret measures of variability and standard deviation and use these measures and the characteristics of the normal distribution to describe and compare data sets.	Students should be able to identify unimodality, symmetry, standard deviation, spread, and the shape of a data curve to determine whether the curve could reasonably be approximated by a normal distribution. Given formulas, student should be able to calculate the standard deviation for a small data set, but calculators ought to be used if there are very many points in the data set. It is important that students be able to describe the characteristics of the normal distribution and identify common examples of data that are and are not reasonably modeled by it. Common examples of distributions that are approximately normal include physical performance measurements (e.g., weightlifting, timed runs), heights, and weights. Apply the Empirical Rule (68–95–99.7 Rule) to approximate the percentage of the population meeting certain criteria in a normal distribution. Example: • Which is more likely to be affected by an outlier in a set of data, the interquartile range or the standard deviation?
M3.4.B Calculate and interpret margin of error and confidence intervals for population proportions.	Students will use technology based on the complexity of the situation. Students use confidence intervals to critique various methods of statistical experimental design, data collection, and data presentation used to investigate important problems, including those reported in public studies. Example: • In 2007, 400 of the 500 10th graders in Local High School passed the WASL. In 2008, 375 of the 480 10th graders passed the test. The Local Gazette headline read “10th Grade WASL Scores Decline in 2008!”

Performance Expectations

Students are expected to:

M3.4.B cont.

Explanatory Comments and Examples

In response, the Superintendent of Local School District wrote a letter to the editor claiming that, in fact, WASL performance was not significantly lower in 2008 than it was in 2007. Who is correct, the Local Gazette or the Superintendent?

Use mathematics to find the margin of error to justify your conclusion. (Formula for the margin of error (E): $E = z_c\sqrt{\frac{p(1-p)}{n}}$; $z_{95} = 1.96$, where n is the sample size, p is the proportion of the sample with the trait of interest, c is the confidence level, and z_c is the multiplier for the specified confidence interval.)

Mathematics 3

M3.5. Core Content: *Three-dimensional geometry* *(Geometry/Measurement)*

Students formulate conjectures about three-dimensional figures. They use deductive reasoning to establish the truth of conjectures or to reject them on the basis of counterexamples. They extend and formalize their work with perimeter, area, surface area, and volume of two- and three-dimensional figures, focusing on mathematical derivations of these formulas and their applications in complex problems. They use properties of geometry and measurement to solve both purely mathematical and applied problems. They also extend their knowledge of distance and angle measurements in a plane to measurements on a sphere.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M3.5.A Describe the intersections of lines in the plane and in space, of lines and planes, and of planes in space.	Example: • Describe all the ways that three planes can intersect in space.
M3.5.B Describe prisms, pyramids, parallelepipeds, tetrahedra, and regular polyhedra in terms of their faces, edges, vertices, and properties.	Examples: • Given the number of faces of a regular polyhedron, derive a formula for the number of vertices. • Describe symmetries of three-dimensional polyhedra and their two-dimensional faces. • Describe the lateral faces that are required for a pyramid to be a right pyramid with a regular base. Describe the lateral faces required for an oblique pyramid that has a regular base.
M3.5.C Analyze cross-sections of cubes, prisms, pyramids, and spheres and identify the resulting shapes.	Examples: • Start with a regular tetrahedron with edges of unit length 1. Find the plane that divides it into two congruent pieces and whose intersection with the tetrahedron is a square. Find the area of the square. (Requires no pencil or paper.) • Start with a cube with edges of unit length 1. Find the plane that divides it into two congruent pieces and whose intersection with the cube is a regular hexagon. Find the area of the hexagon. • Start with a cube with edges of unit length 1. Find the plane that divides it into two congruent pieces and whose intersection with the cube is a rectangle that is not a face and contains four of the vertices. Find the area of the rectangle. • Which has the larger area, the above rectangle or the above hexagon?

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
M3.5.D Apply formulas for surface area and volume of three-dimensional figures to solve problems.	Problems include those that are purely mathematical as well as those that arise in applied contexts. Three-dimensional figures include right and oblique prisms, pyramids, cylinders, cones, spheres, and composite three-dimensional figures. Examples: • As Pam scooped ice cream into a cone, she began to formulate a geometry problem in her mind. If the ice cream was perfectly spherical with diameter 2.25” and sat on a geometric cone that also had diameter 2.25” and was 4.5” tall, would the cone hold all the ice cream as it melted (without her eating any of it)? She figured the melted ice cream would have the same volume as the unmelted ice cream. Find the solution to Pam’s problem and justify your reasoning. • A rectangle is 5 inches by 10 inches. Find the volume of a cylinder that is generated by rotating the rectangle about the 10-inch side.
M3.5.E Predict and verify the effect that changing one, two, or three linear dimensions has on perimeter, area, volume, or surface area of two- and three-dimensional figures.	The emphasis in high school should be on verifying the relationships between length, area, and volume and on making predictions using algebraic methods. Examples: • What happens to the volume of a rectangular prism if four parallel edges are doubled in length? • The ratio of a pair of corresponding sides in two similar triangles is 5:3. The area of the smaller triangle is 108 in^2. What is the area of the larger triangle?
M3.5.F Analyze distance and angle measures on a sphere and apply these measurements to the geometry of the earth.	Examples: • Use a piece of string to measure the distance between two points on a ball or globe; verify that the string lies on an arc of a great circle. • On a globe, show with examples why airlines use polar routes instead of flying due east from Seattle to Paris. • Show that the sum of the angles of a triangle on a sphere is greater than 180 degrees.

Mathematics 3

***M3.6. Core Content:** Algebraic properties* *(Numbers, Algebra)*

Students continue to use variables and expressions to solve both purely mathematical and applied problems, and they broaden their understanding of the real number system to include complex numbers. Students extend their use of algebraic techniques to include manipulations of expressions with rational exponents, operations on polynomials and rational expressions, and solving equations involving rational and radical expressions.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M3.6.A Explain how whole, integer, rational, real, and complex numbers are related, and identify the number system(s) within which a given algebraic equation can be solved.	Example: • Within which number system(s) can each of the following be solved? Explain how you know. — $3x + 2 = 5$ — $x^2 = 1$ — $x^2 = \frac{1}{4}$ — $x^2 = 2$ — $x^2 = -2$ — $\frac{x}{7} = \pi$
M3.6.B Use the laws of exponents to simplify and evaluate numeric and algebraic expressions that contain rational exponents.	Examples: • Convert the following from a radical to exponential form or visa versa. — $24^{\frac{1}{3}}$ — $\sqrt[5]{16}$ — $\sqrt{x^2+1}$ — $\frac{x^2}{\sqrt{x}}$ • Evaluate $x^{-3/2}$ for $x = 27$.
M3.6.C Add, subtract, multiply, and divide polynomials.	Write algebraic expressions in equivalent forms using algebraic properties to perform the four arithmetic operations with polynomials. Students should recognize that expressions are essentially sums, products, differences, or quotients. For example, the sum $2x^2 + 4x$ can be written as a product, $2x(x + 2)$.

Performance Expectations *Students are expected to:*	Explanatory Comments and Examples
M3.6.C cont.	Examples: • $(3x^2 - 4x + 5) + (-x^2 + x - 4) + (2x^2 + 2x + 1)$ • $(2x^2 - 4) - (x^2 + 3x - 3)$ • $\frac{2x^2}{9} \bullet \frac{6}{2x^4}$ • $\frac{x^2 - 2x - 3}{x+1}$
M3.6.D Add, subtract, multiply, divide, and simplify rational and more general algebraic expressions.	In the same way that integers were extended to fractions, polynomials are extended to rational expressions. Students must be able to perform the four basic arithmetic operations on more general expressions that involve exponentials. The binomial theorem is useful when raising expressions to powers, such as $(x + 3)^5$. Examples: • $\frac{x+1}{(x+1)^2} - \frac{3x-3}{x^2-1}$ • Divide $\frac{(x+2)^{3/2}}{x+1}$ by $\frac{x+2}{x^2-1}$

Mathematics 3

M3.7. Additional Key Content *(Geometry/Measurement)*

Students formulate conjectures about circles. They use deductive reasoning to establish the truth of conjectures or to reject them on the basis of counterexamples. Students explain their reasoning using precise mathematical language and symbols. They apply their knowledge of geometric figures and their properties to solve a variety of both purely mathematical and applied problems.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M3.7.A Know, prove, and apply basic theorems relating circles to tangents, chords, radii, secants, and inscribed angles.	Examples: • Given a line tangent to a circle, know and explain that the line is perpendicular to the radius drawn to the point of tangency. • Prove that two chords equally distant from the center of a circle are congruent. • Prove that if one side of a triangle inscribed in a circle is a diameter, then the triangle is a right triangle. • Prove that if a radius of a circle is perpendicular to a chord of a circle, then the radius bisects the chord.
M3.7.B Determine the equation of a circle that is described geometrically in the coordinate plane and, given equations for a circle and a line, determine the coordinates of their intersection(s).	Examples: • Write an equation for a circle with a radius of 2 units and center at (1, 3). • Given the circle $x^2 + y^2 = 4$ and the line $y = x$, find the points of intersection. • Write an equation for a circle given a line segment as a diameter. • Write an equation for a circle determined by a given center and tangent line.
M3.7.C Explain and perform constructions related to the circle.	Students perform constructions using straightedge and compass, paper folding, and dynamic geometry software. What is important is that students understand the mathematics and are able to justify each step in a construction. Example: • In each case, explain why the constructions work: a. Construct the center of a circle from two chords. b. Construct a circumscribed circle for a triangle. c. Inscribe a circle in a triangle.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	

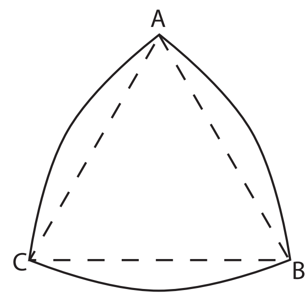
M3.7.D Derive and apply formulas for arc length and area of a sector of a circle.

Example:

- Find the area and perimeter of the Reuleaux triangle below.

  The Reuleaux triangle is constructed with three arcs. The center of each arc is located at the vertex of an equilateral triangle. Each arc extends between the two opposite vertices of the equilateral triangle.

  The figure below is a Reuleaux triangle that circumscribes equilateral triangle ABC. ΔABC has side length of 5 inches. $\overset{\frown}{AB}$ has center C, $\overset{\frown}{BC}$ has center A, and $\overset{\frown}{CA}$ has center B, and all three arcs have the same radius equal to the length of the sides of the triangle.



Mathematics 3

M3.8. Core Processes: *Reasoning, problem solving, and communication*

Students formalize the development of reasoning in Mathematics 3 as they use algebra, geometry, and statistics to make and defend generalizations. They justify their reasoning with accepted standards of mathematical evidence and proof, using correct mathematical language, terms, and symbols in all situations. They extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to mathematical and applied situations. Students formalize a coherent problem-solving process in which they analyze the situation to determine the question(s) to be answered, synthesize given information, and identify implicit and explicit assumptions that have been made. They examine their solution(s) to determine reasonableness, accuracy, and meaning in the context of the original problem. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an increasing amount of information is presented in quantitative ways and more and more occupations and fields of study rely on mathematics.

Performance Expectations

Students are expected to:

M3.8.A Analyze a problem situation and represent it mathematically.

M3.8.B Select and apply strategies to solve problems.

M3.8.C Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.

M3.8.D Generalize a solution strategy for a single problem to a class of related problems and apply a strategy for a class of related problems to solve specific problems.

M3.8.E Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.

M3.8.F Summarize mathematical ideas with precision and efficiency for a given audience and purpose.

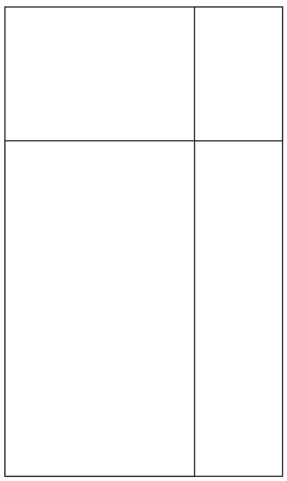
M3.8.G Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.

M3.8.H Use inductive reasoning and the properties of numbers to make conjectures, and use deductive reasoning to prove or disprove conjectures.

Explanatory Comments and Examples

Examples:

- Show that $\sqrt{a+b} \neq \sqrt{a}+\sqrt{b}$, for all positive real values of *a* and *b*.
- Show that the product of two odd numbers is always odd.
- Leo is painting a picture on a canvas that measures 32 inches by 20 inches. He has divided the canvas into four different rectangles, as shown in the diagram.



He would like the upper right corner to be a rectangle that has a length 1.6 times its width. Leo wants the area of the larger rectangle in the lower left to be at least half the total area of the canvas.

Describe all the possibilities for the dimensions of the upper right rectangle to the nearest hundredth, and explain why the possibilities are valid.

Performance Expectations

Students are expected to:

M3.8 cont.

Explanatory Comments and Examples

If Leo uses the largest possible dimensions for the smaller rectangle:

- What will the dimensions of the larger rectangle be?
- Will the larger rectangle be similar to the rectangle in the upper right corner? Why or why not?
- Is the original canvas similar to the rectangle in the upper right corner?

(A rectangle whose length and width are in the ratio $\frac{1+\sqrt{5}}{2}$ (approximately equal to 1.6) is called a "golden rectangle" and is often used in art and architecture.)

- A relationship between variables can be represented with a table, a graph, an equation, or a description in words.
  - How can you decide from a table whether a relationship is linear, quadratic, or exponential?
  - How can you decide from a graph whether a relationship is linear, quadratic, or exponential?
  - How can you decide from an equation whether a relationship is linear, quadratic, or exponential?

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Washington Educators and Community Leaders

Dana Anderson, Stanwood-Camano School District
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Art Mabbott, Seattle Schools
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David Thielk, Central Kitsap School District
Johnnie Tucker, retired teacher
Kimberly Vincent, Washington State University
Virginia Warfield, University of Washington
Sharon Young, Seattle Pacific University

Dana Center Facilitators

P. Uri Treisman
Cathy Seeley
Susan Hudson Hull

National Consultants

Mary Altieri, Consultant (retired teacher)
Angela Andrews, National-Louis University
Diane Briars, Pittsburgh Schools (retired)
Cathy Brown, Oregon Department of Education (retired)
Dinah Chancellor, Consultant
Philip Daro, Consultant
Bill Hopkins, Dana Center
Barbara King, Dana Center
Kurt Krieth, University of California at Davis
Bonnie McNemar, Consultant
David D. Molina, Consultant
Susan Eddins, Illinois Math and Science Academy (retired)
Wade Ellis, West Valley College, CA (retired)
Margaret Myers, The University of Texas at Austin
Lynn Raith, Pittsburgh Schools (retired)
Jane Schielack, Texas A&M University
Carmen Whitman, Consultant

OSPI Project Support

George W. Bright, Special Assistant to the Superintendent
Greta Bornemann, Teaching and Learning
Barbara Chamberlain, Interim Director
Larry Davison, Math Helping Corps Administrator
Lexie Domaradzki, Assistant Superintendent, Teaching and Learning
Ron Donovan, Teaching & Learning
Dorian "Boo" Drury, Teaching & Learning
Lynda Eich, Assessment
Karen Hall, Assessment
Robert Hodgman, Assessment
Mary Holmberg, Assessment
Anton Jackson, Assessment
Yoonsun Lee, Assessment
Karrin Lewis, Teaching & Learning
Jessica Vavrus, Teaching and Learning
Joe Willhoft, Assistant Superintendent, Assessment

Strategic Teaching Reviewers

Linda Plattner
Andrew Clark
W. Stephen Wilson